UNIVERSITÉ LIBRE DE BRUXELLES, UNIVERSITÉ D'EUROPE

Geometrically nonlinear finite element modelling of linear elastic truss structrures

Péter Z. Berke

1.3. Linear bar finite element

Inspired and adapted from the 'Calcul des structures par ordinateur' course of Profs. Guy Warzée and Philippe Bouillard at the ULB

ULB

2D linear elastic bar element

Lab: Understanding of main points in the code

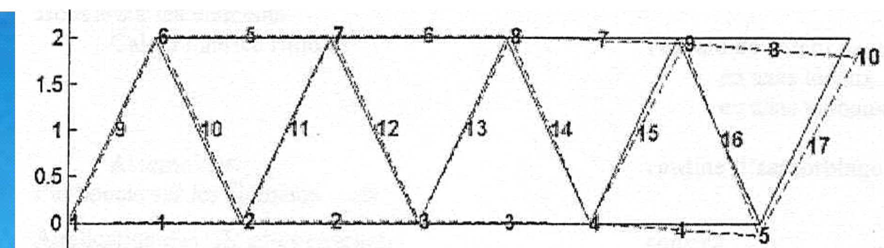
Complete missing element relationships

2D linear elastic bar element

Problem statement

Determine the displacements of structures at equilibrium

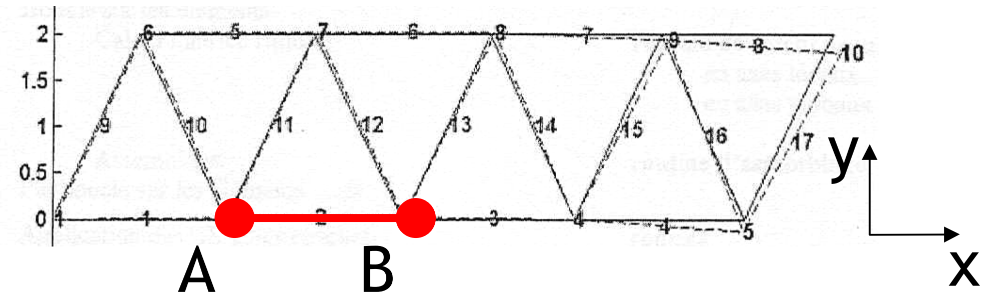
Small displacements and deformations, linear elastic material

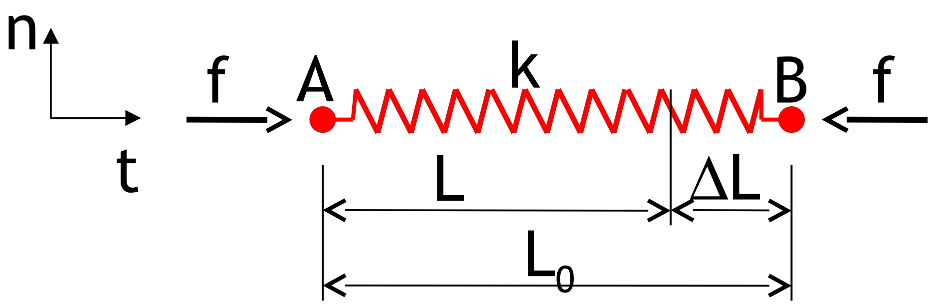


Stadium of Geneva

2D linear elastic bar element

Relationships in local element axes (n, t)





Linear strain measure

$$\epsilon = \frac{\Delta L}{L_0}$$

+

Linear elasticity

$$\sigma = \epsilon\, E$$

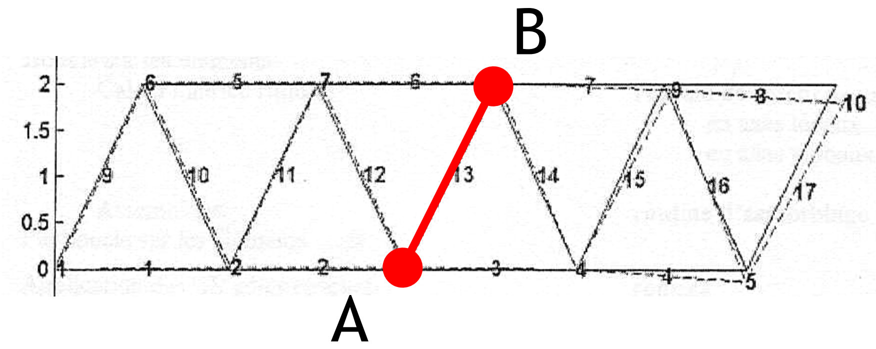
⇒

$\{q\}_e$ displacements

$\{f\}_e$ forces

$$\frac{E\,A}{L_0}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_A \\ u_B \end{Bmatrix} = \begin{Bmatrix} f_A \\ f_B \end{Bmatrix}$$

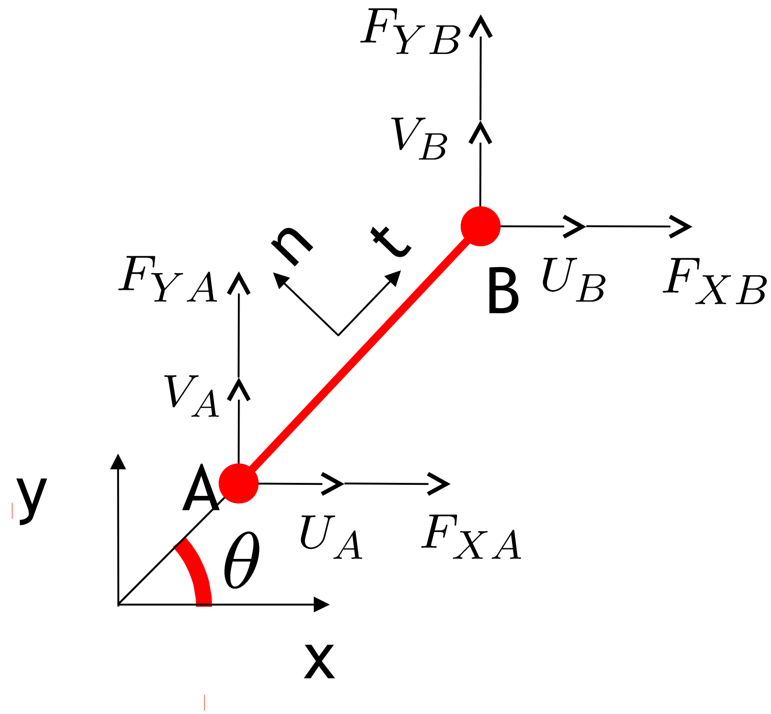
2D linear elastic bar element



Relationships in structural axes (x, y)

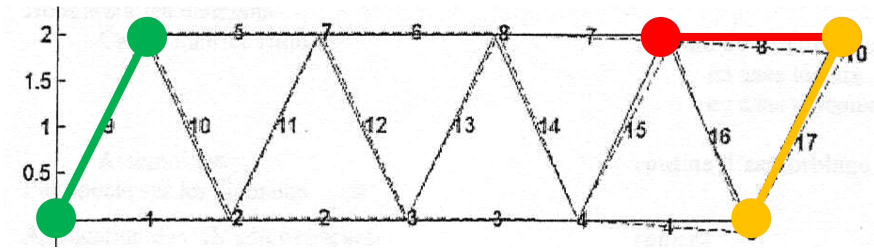
$$\left\{\begin{aligned} \{q\}_e &= [R]\,\{Q\}_e \\ \{f\}_e &= [R]\,\{F\}_e \\ [K]_e &= [R]^t\,[k]_e\,[R] \end{aligned}\right.$$

$$R(\theta) = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix}$$

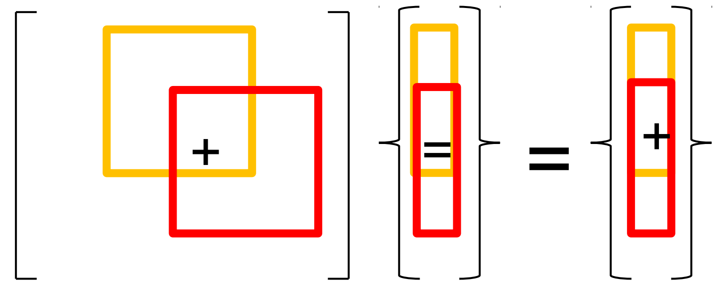


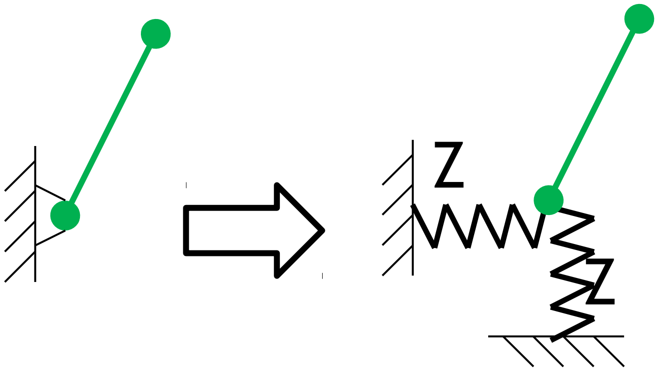
$$[K]_e\,\{Q\}_e = \{F\}_e$$ in (x, y)

2D linear elastic bar element



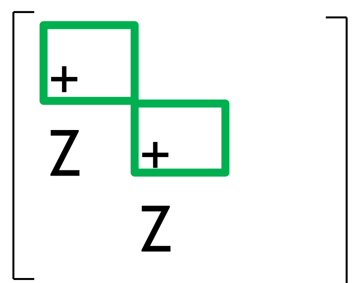
Assembly of the system in structural axes (x, y)





with Z a high spring stiffness

Boundary conditions



$$= [K]_{BC}$$

Use of the element in the NL code

- Incremental loop (for)
  - Initialize the residual
  - Iteration loop (while residual > tolerance)
    - Assembly of the tangent stiffness
    - Elimination of the prescribed and dependent dof
    - Solve the system
    - Substitute prescribed and dependent dof
    - Compute internal forces
    - Compute new residual
  - End of iteration loop
  - Save converged displacements
- End of incremental loop