Geometrically nonlinear finite element modelling of linear elastic truss structrures

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1.1. Newton-Raphson solution of nonlinear (system of) equations

Inspired and adapted from the 'Nonlinear Modeling of Structures' course of Prof. Thierry J. Massart at the ULB

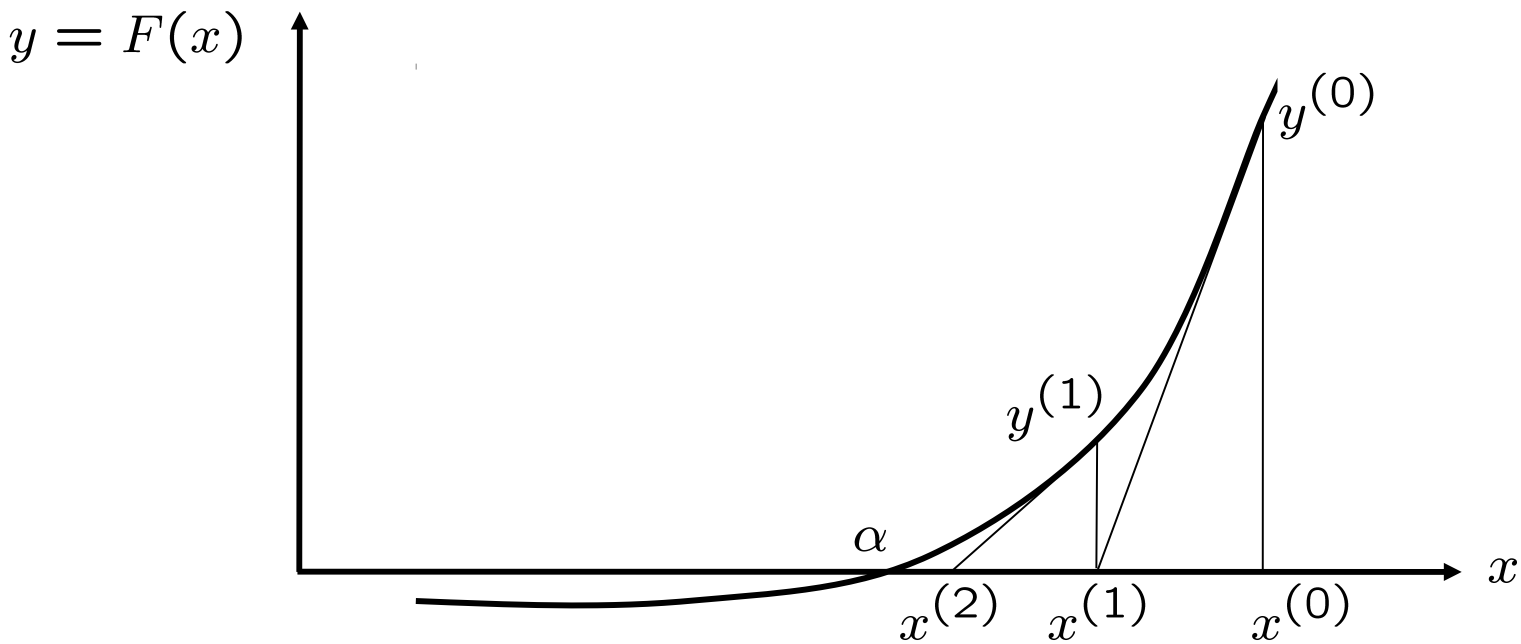
Outline

Newton-Raphson procedure for 1 NL equation

N-R in the structure of a NL FE code

Lab: Solve 1 NL equation using N-R in MatLab

Find a new approximation based on an initial value and the slope of the function at this point



Newton-Raphson procedure

Iterative scheme

Re-write $F(x) = 0$ under the form $x = f(x)$

Construct a series of successive approximations

$$x^{(1)} = f(x^{(0)})$$

$$x^{(k)} = f(x^{(k-1)})$$

Newton-Raphson approximation

$$x^{(1)} = x^{(0)} - \frac{F(x^{(0)})}{F'(x^{(0)})}$$

$$x^{(k+1)} = x^{(k)} - \frac{F(x^{(k)})}{F'(x^{(k)})}$$

Requires the knowledge of the function derivative

Quadratic local convergence, <u>if the derivative is right</u>

This last point is CRUCIAL for a proper convergence!

Newton-Raphson procedure

Interpretation from a series development

Assume a first approximation is available $x^{(k)}$

Express the value of the function as a first order development

$$F(x^{(k+1)}) = F(x^{(k)}) + F'(x^{(k)})(x^{(k+1)} - x^{(k)}) + \frac{F''(x^{(k)})}{2!}(x^{(k+1)} - x^{(k)})^2 + \ldots$$

If this new value has to vanish (to find the root)

$$F(x^{(k+1)}) = 0$$

Using the first order development, a new approximation is

$$F(x^{(k)}) + F'(x^{(k)})(x^{(k+1)} - x^{(k)}) \approx 0$$

$$x^{(k+1)} = x^{(k)} - \frac{F(x^{(k)})}{F'(x^{(k)})}$$

Newton-Raphson procedure

Inremental-iterative loop

increment P_N

initialize residual $\left(P_{N+1} - P\left(w\right)\right)$

while residual too large

compute $\left[K_t\left(w\right)\right]^{(i)}$

$$[K_t]^{(i)}\,\delta w^{(i+1)} = \left(P_{N+1} - P\left(w^{(i)}\right)\right)$$

compute $P\left(w^{(i+1)}\right)$

compute new residual

end of iteration loop

save converged results

end of incremental loop

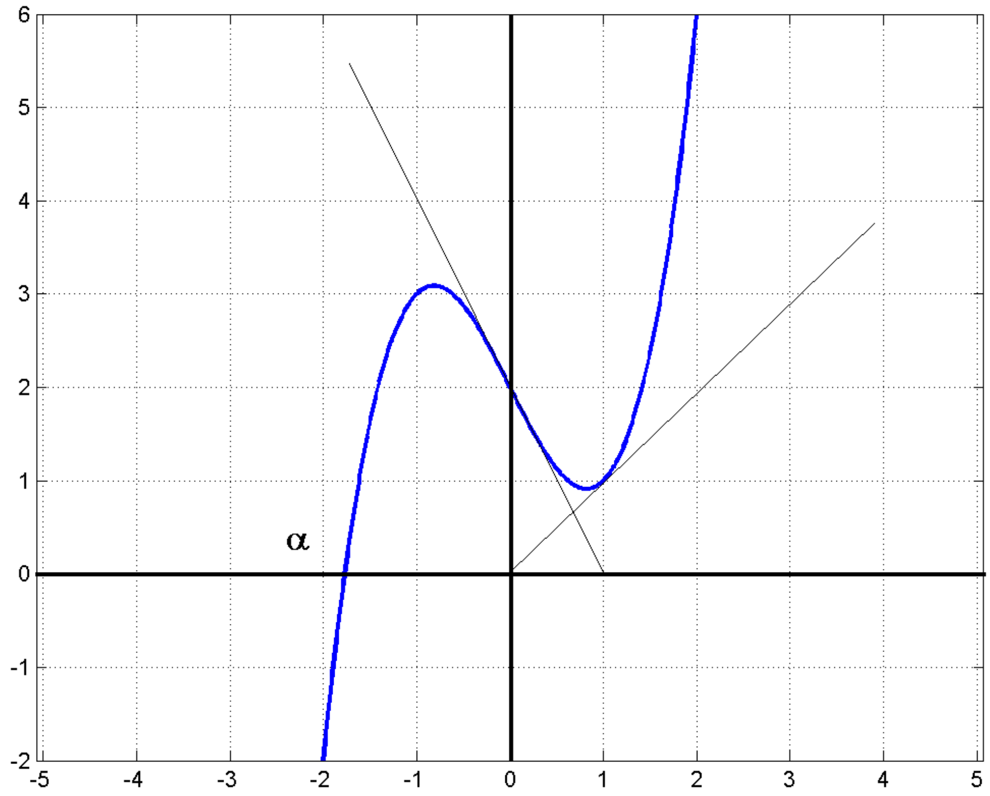
Newton-Raphson procedure

Potential shortcomings

Vanishing derivatives

Deadlocks between particular points

$$F(x) = x^3 - 2x + 2$$



Newton-Raphson procedure

For a system of equations

$$F_1(x_1, ..., x_n) = 0$$
$$\cdots$$
$$F_n(x_1, ..., x_n) = 0$$

The iterative scheme becomes

Initial approximation $\left\{x^{(0)}\right\} = \left\{x_1^{(0)}, ..., x_n^{(0)}\right\}^T$

A new approximation is found by solving

$$\left[J_F\left(\left\{x^{(k)}\right\}\right)\right]\left(\left\{x^{(k+1)}\right\} - \left\{x^{(k)}\right\}\right) = -\left\{F\left(\left\{x^{(k)}\right\}\right)\right\}$$

Jacobian matrix

Newton-Raphson procedure

Flowchart

Define a set of successive loading states $F_{ext,n}$

Loop on the loading states (steps or increments)

Formulate the problem for the step $n \rightarrow n+1$

Find q_{n+1} such that $F_{int}(q_{n+1}) - F_{ext,n+1} = 0$

With as first approximation $q_{n+1}^{(0)} = q_n$

Iterate until a precision threshold is reached with

$$F_{int,n}^{(k)} = F_{int}\left(q_{n+1}^{(k)}\right)$$

$$q_{n+1}^{(k+1)} = q_{n+1}^{(k)} - \left(\frac{\partial F_{int}}{\partial q}\right)_{q_{n+1}^{(k)}}^{-1} \left(F_{ext,n+1} - F_{int,n}^{(k)}\right)$$

End of step

Newton-Raphson procedure

Graphical interpretation