1st International Workshop PROPEC 15 years – Lecture #7

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Part I - Development of an advanced FE analysis program for the simulation of steel, concrete and composite structures under fire action

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Outline

- Objectives
- Heat transfer analysis
- Cross section behaviour
- Beam column finite element
- Shell finite element
- Examples

Objectives

- Extend an object-oriented, C++ generic FE program to analyze structures under fire action, comprising:
  - Heat transfer analysis at the cross section level to evaluate temperature profile
  - Proper constitutive models for concrete, steel and protection materials
  - Nonlinear analysis of three-dimensional framed assemblies under fire
  - Nonlinear analysis of reinforced concrete and composite slabs under fire
  - Analysis of semi-rigid connections under fire action

Heat transfer analysis

- The heat transfer analysis is carried out in the cross section level to evaluate the temperature profile and its effects on the resistance of the section.
- Numerical schemes implemented are based on finite difference (FD) and finite element (FE) methods.
- Material properties under high temperatures are taken from design codes such as the Eurocodes 2 and 4.
- The resultant forces are obtained using the concept of *effective strain*.

Heat transfer analysis

Heat transfer equation for 3-D problems

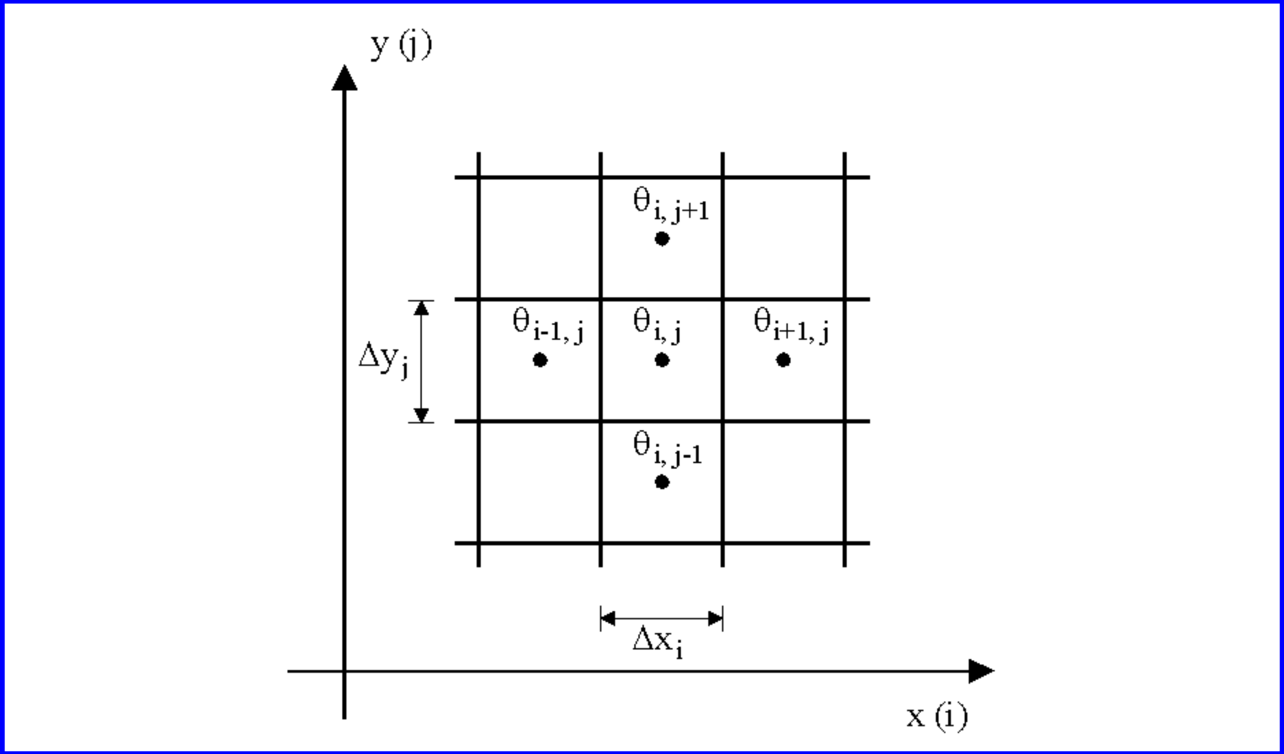
$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial \theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y \frac{\partial \theta}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda_z \frac{\partial \theta}{\partial z}\right)+\dot{\psi}=\rho c \frac{\partial \theta}{\partial t}$$

θ = temperature; λ= thermal conductivity;
ψ = generated heat; ρ = mass per unit volume;
c = specific heat; t = time

Boundary conditions include prescribed temperatures, heat fluxes (convection and radiation).

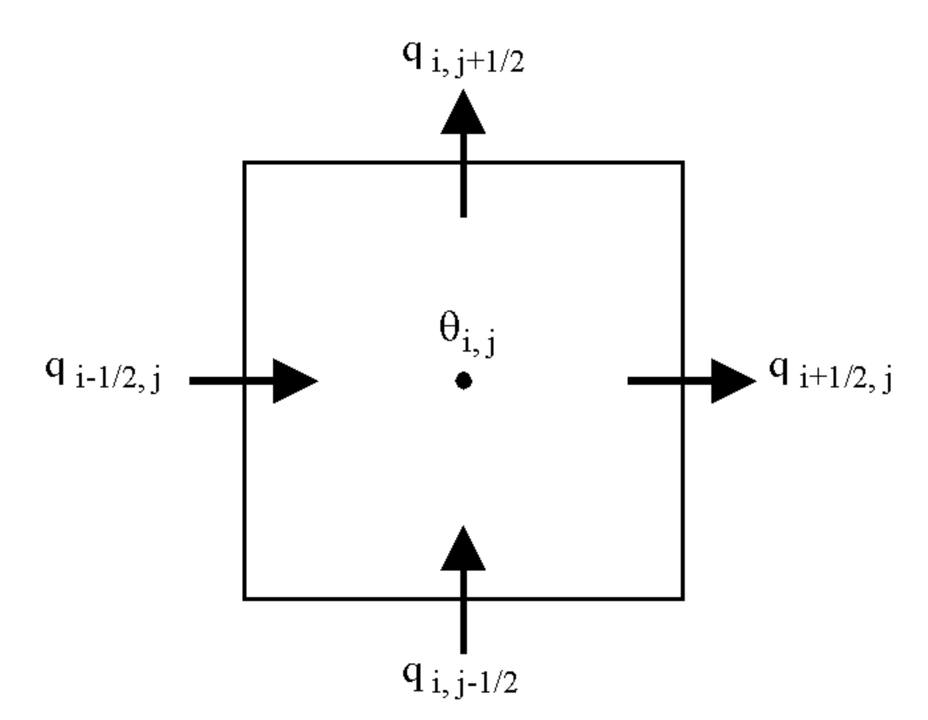
Finite difference scheme

Finite difference mesh with temperatures defined at the center of each grid square



Finite difference scheme

Heat flux through the 4 sides of a single element in the grid



$$q_{i-\frac{1}{2},j} = k_{i-\frac{1}{2},j}\left(\theta_{i-1,j} - \theta_{i,j}\right)$$

$$\mathrm{q}_{\mathrm{i},\mathrm{j}-\frac{1}{2}} = \mathrm{k}_{\mathrm{i},\mathrm{j}-\frac{1}{2}}\left(\theta_{\mathrm{i},\mathrm{j}-1} - \theta_{\mathrm{i},\mathrm{j}}\right)$$

Finite difference scheme

Energy balance for an element of the grid

$$c_{v\,i,j}\Delta x_i \Delta y_j\left(\theta^n_{i,j}-\theta_{i,j}\right)=\left(q_{i-\frac{1}{2},j}-q_{i+\frac{1}{2},j}+q_{i,j-\frac{1}{2}}-q_{i,j+\frac{1}{2}}\right)\Delta t$$

Temperature update on each element

$$\theta^n_{i,j}=\theta_{i,j}+\frac{\Delta t}{c_{v\,i,j}\Delta x_i \Delta y_j}\left(q_{i-\frac{1}{2},j}-q_{i+\frac{1}{2},j}+q_{i,j-\frac{1}{2}}-q_{i,j+\frac{1}{2}}\right)$$

To ensure stability:

$$\Delta t<\frac{c_{v\,i,j}\Delta x_i \Delta y_j}{k_{i-\frac{1}{2},j}+k_{i+\frac{1}{2},j}+k_{i,j-\frac{1}{2}}+k_{i,j+\frac{1}{2}}}$$

Finite element scheme

After spatial discretization and standard FE interpolation, one gets

$$\boxed{\mathbf{K}\boldsymbol{\theta} + \mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{F}}$$

where:

$$K_{\ell m} = K_{\ell m,1} + K_{\ell m,2} = \sum_{e=1}^{E} \int_{\Omega^e} \nabla N_\ell \lambda \nabla N_m d\Omega^e + \sum_{e=1}^{H} \int_{\Gamma^e_{\varphi_{cr}}} \alpha_{cr} N_\ell N_m d\Gamma^e_{\varphi_{cr}}$$

$$\mathrm{C}_{\ell \mathrm{m}} = \sum_{\mathrm{e}=1}^{\mathrm{E}} \int_{\Omega^{\mathrm{e}}} \rho \mathrm{c} \mathrm{N}_\ell \mathrm{N}_{\mathrm{m}} \mathrm{d}\Omega^{\mathrm{e}}$$

$$\mathrm{F}_\ell = \sum_{\mathrm{e}=1}^{\mathrm{E}} \int_{\Omega^{\mathrm{e}}} \mathrm{N}_\ell \psi \mathrm{d}\Omega^{\mathrm{e}} - \sum_{\mathrm{e}=1}^{\mathrm{Q}} \int_{\Gamma^{\mathrm{e}}_{\varphi_{\mathrm{p}}}} \mathrm{N}_\ell \varphi_{\mathrm{p}} \mathrm{d}\Gamma^{\mathrm{e}}_{\varphi_{\mathrm{p}}} + \sum_{\mathrm{e}=1}^{\mathrm{H}} \int_{\Gamma^{\mathrm{e}}_{\varphi_{\mathrm{cr}}}} \mathrm{N}_\ell \alpha_{\mathrm{cr}} \theta_{\mathrm{g}} \mathrm{d}\Gamma^{\mathrm{e}}_{\varphi_{\mathrm{cr}}}$$

Finite element scheme

Time integration scheme: $\boldsymbol{\theta}_{n+\delta} = \boldsymbol{\theta}_n + \frac{\delta\Delta t\left(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n\right)}{\Delta t} \quad \text{com} \quad 0 \le \delta \le 1$

Assuming linear temperature variation $\frac{\partial \boldsymbol{\theta}_{n+\delta}}{\partial t} = \frac{\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n}{\Delta t}$

We arrive at the recurrence formula $\hat{\mathbf{K}}_{\mathrm{n+\delta}} \boldsymbol{\theta}_{\mathrm{n+\delta}} = \hat{\mathbf{F}}_{\mathrm{n+\delta}}$

And the new temperatures are $\boldsymbol{\theta}_{\mathrm{n+1}} = \frac{1}{\delta} \boldsymbol{\theta}_{\mathrm{n+\delta}} + \left(1 - \frac{1}{\delta}\right) \boldsymbol{\theta}_{\mathrm{n}}$

Finite element scheme

In general, the material thermal properties are temperature-dependent, resulting in a nonlinear system of equations:

$$\mathbf{K}(\theta, t)\boldsymbol{\theta}(t) + \mathbf{C}(\theta, t)\dot{\boldsymbol{\theta}}(t) = \mathbf{F}(\theta, t)$$

Solution of this system involves proper time integration schemes along with an iterative process. The simplest option employs the simple iteration

$$\boldsymbol{\theta}_{n+\delta}^{i+1} = \left[\hat{\mathbf{K}}_{n+\delta}^{i}\right]^{-1} \hat{\mathbf{F}}_{n+\delta}^{i}$$

Comments about the methods

- FDM is extremely fast and provides good precision for the purposes of this work, but is not suitable to model complex boundaries.
- FEM is computationally expensive but handles complex geometries and boundary conditions.
- Some adjustments to the FE scheme were made to gain speed.

Examples of heat transfer analysis

Example 1. Concrete-filled circular tube (Lie 1994)

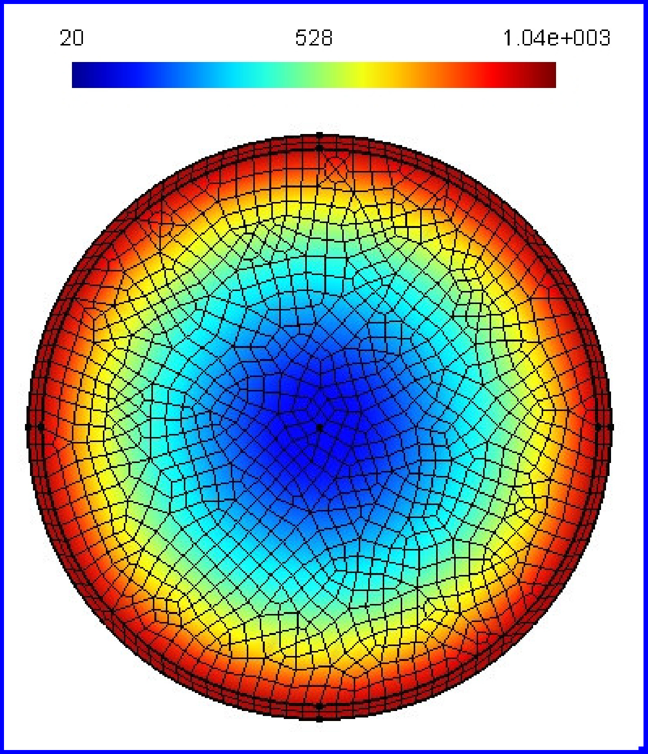
- external diameter 273 mm
- thickness 6,35 mm
- moisture content for concrete taken as 10%

Lie, T.T. (1994). Fire Resistance of Circular Steel Columns Filled with Bar-Reinforced Concrete. Journal of Structural Engineering, 120, 1489-1509.

Examples of heat transfer analysis

Example 1. Concrete-filled circular tube (Lie 1994)

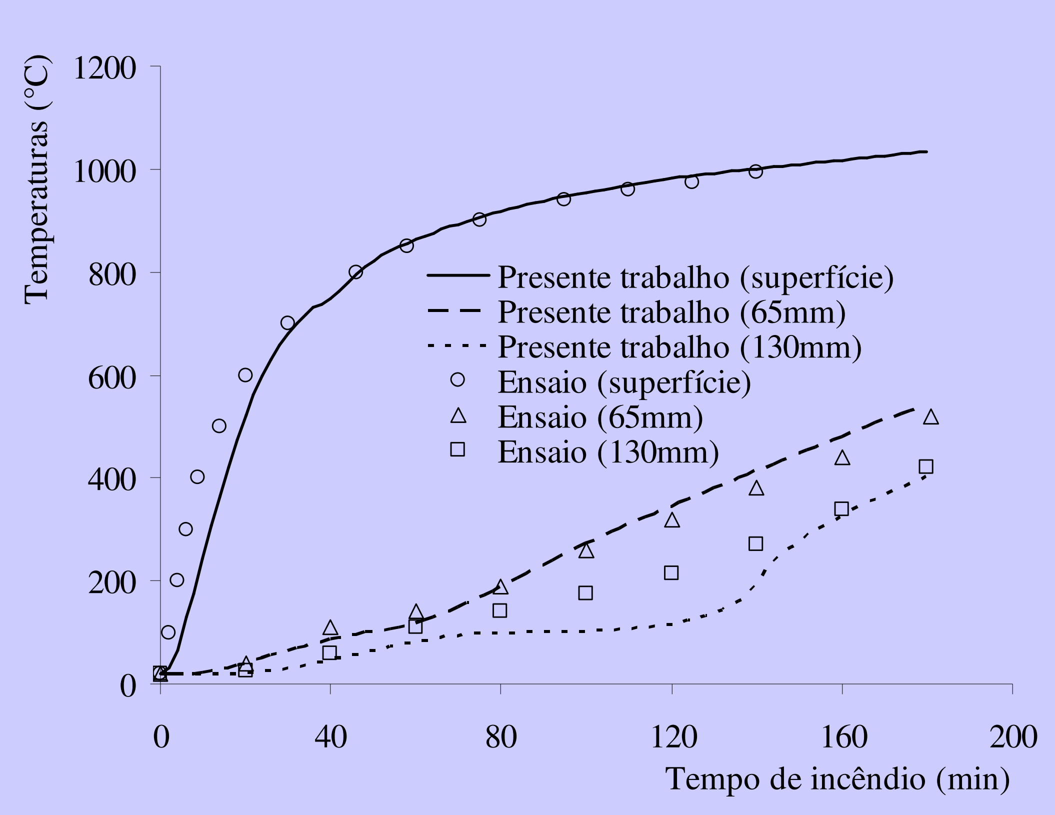
Temperature distribution for 60 min of fire exposure



Examples of heat transfer analysis

Example 1. Concrete-filled circular tube (Lie 1994)

Temperature-time results for 3 different points of the section



Examples of heat transfer analysis

Example 2. Concrete-encased composite column (Huang et al. 2007)

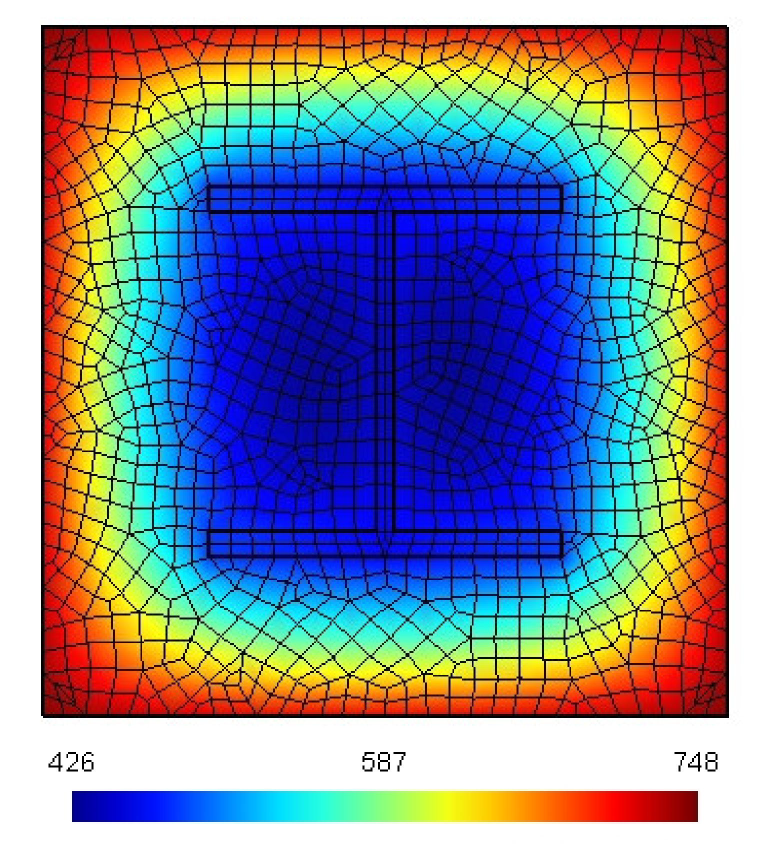
- UC 152x152x37 with 300x300 mm^2 concrete cover
- EN 1994-1-2:2005 with moisture 8% of concrete weight
- upper limit for concrete conductivity

Huang, Z.F., Tan, K.H., Phng, G.H. (2007). Axial Restraint Effects on the Fire Resistance of Composite Columns Encasing I-Section Steel. J. Constr. Steel Res., 63, 437-447.

Examples of heat transfer analysis

Example 2. Concrete-encased composite column (Huang et al. 2007)

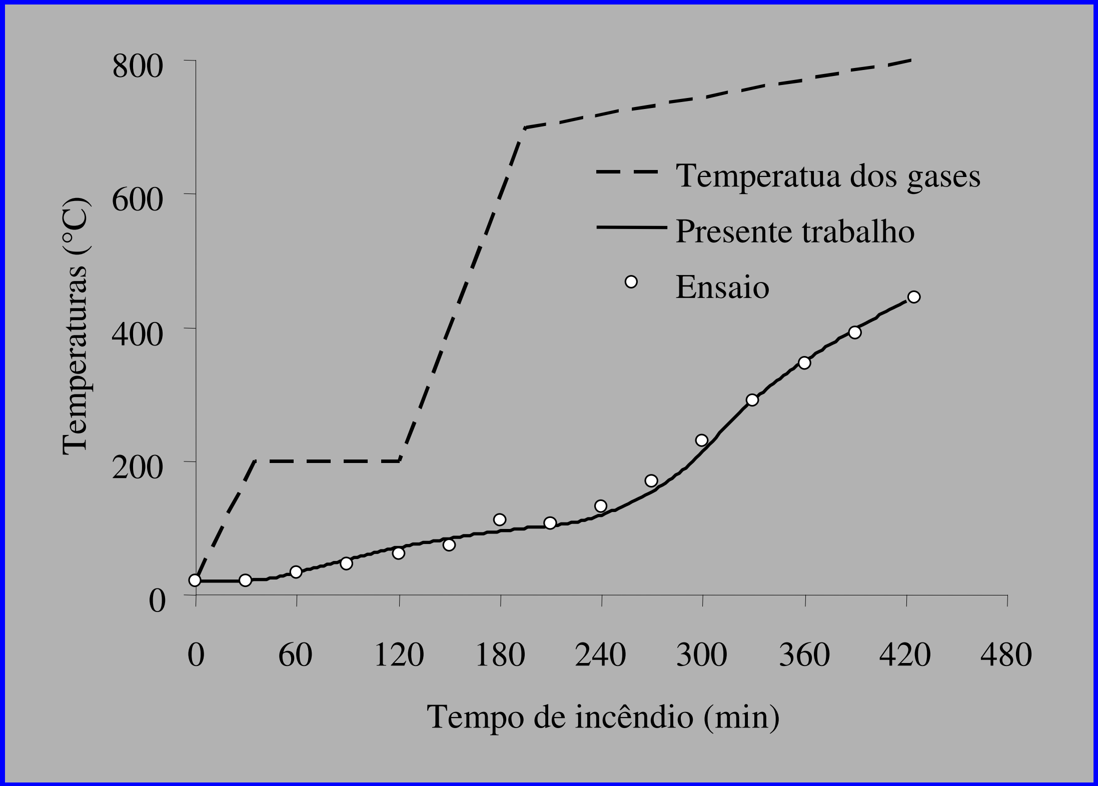
Temperature distribution for 420 min of fire exposure



Examples of heat transfer analysis

Example 2. Concrete-encased composite column (Huang et al. 2007)

Results for temperature, for numerical and experimental analysis

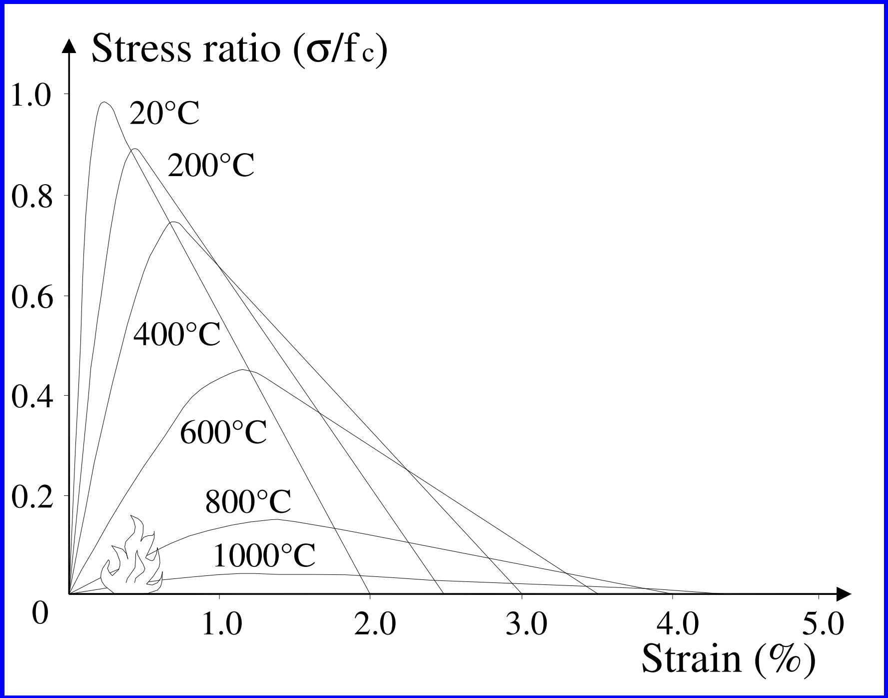


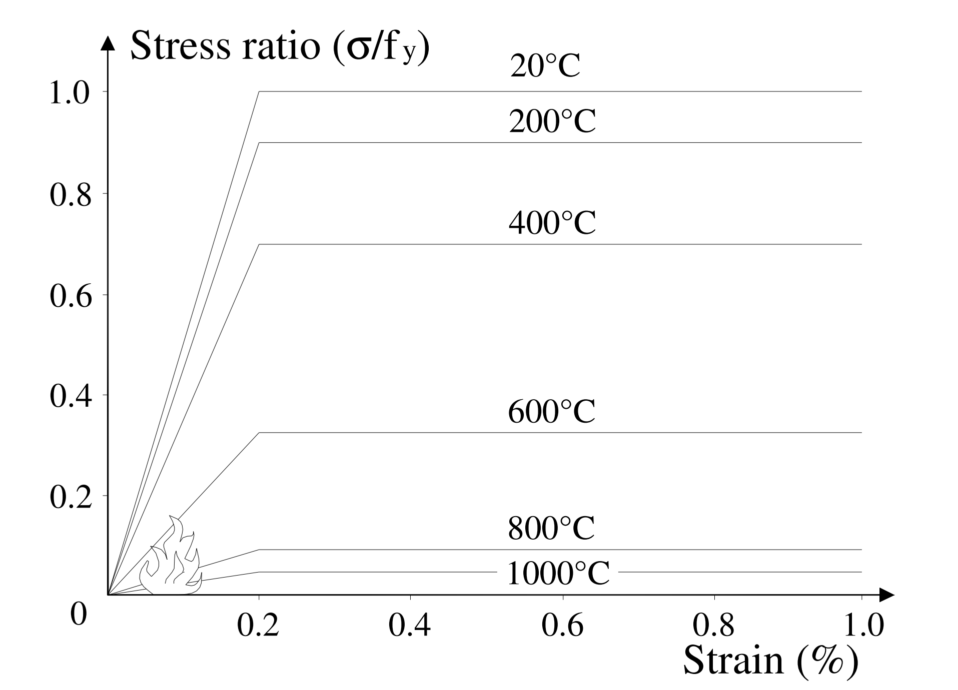
Beam-column element formulation

- With the temperature distribution it is possible to evaluate the cross section resistant forces (N, Mx, My) and tangent moduli. This will allow the evaluation of the cross section internal force and tangent stiffness, which is necessary for the development of the nonlinear finite element.
- The material degradation is taken into account by the modification of the stress-strain relationships provided by the Eurocodes.

Beam-column element formulation

Stress-strain relations for concrete and steel at elevated temperatures





Beam-column element formulation

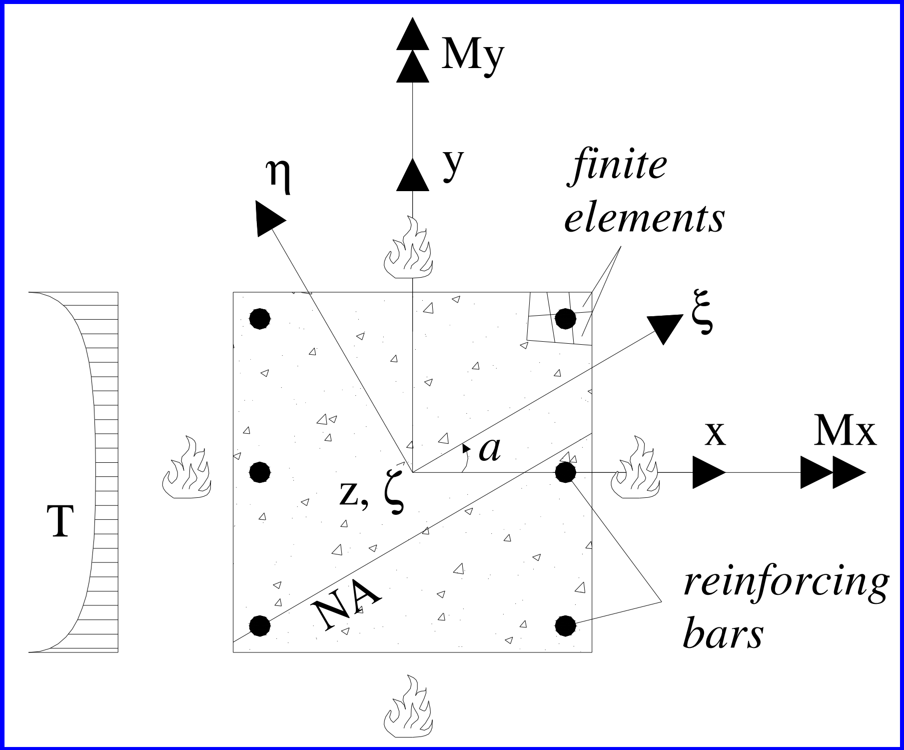
Mechanical (stress-inducing) strain at a point in the cross section

$$\varepsilon_{mech}(x,y) = \varepsilon_{total} - \varepsilon_{th} = \varepsilon_o + k_x y - k_y x - \varepsilon_{th}$$

The concrete stress-strain relationship relates stress to mechanical strain and is supposed to include effects of creep and transient strains (load-induced thermal strains). This is still much open to debate as there are several proposals for these relationships.

Beam-column element formulation

Using the same mesh employed for thermal analysis it is possible to evaluate cross section resistant forces and tangent moduli:



Beam-column element formulation

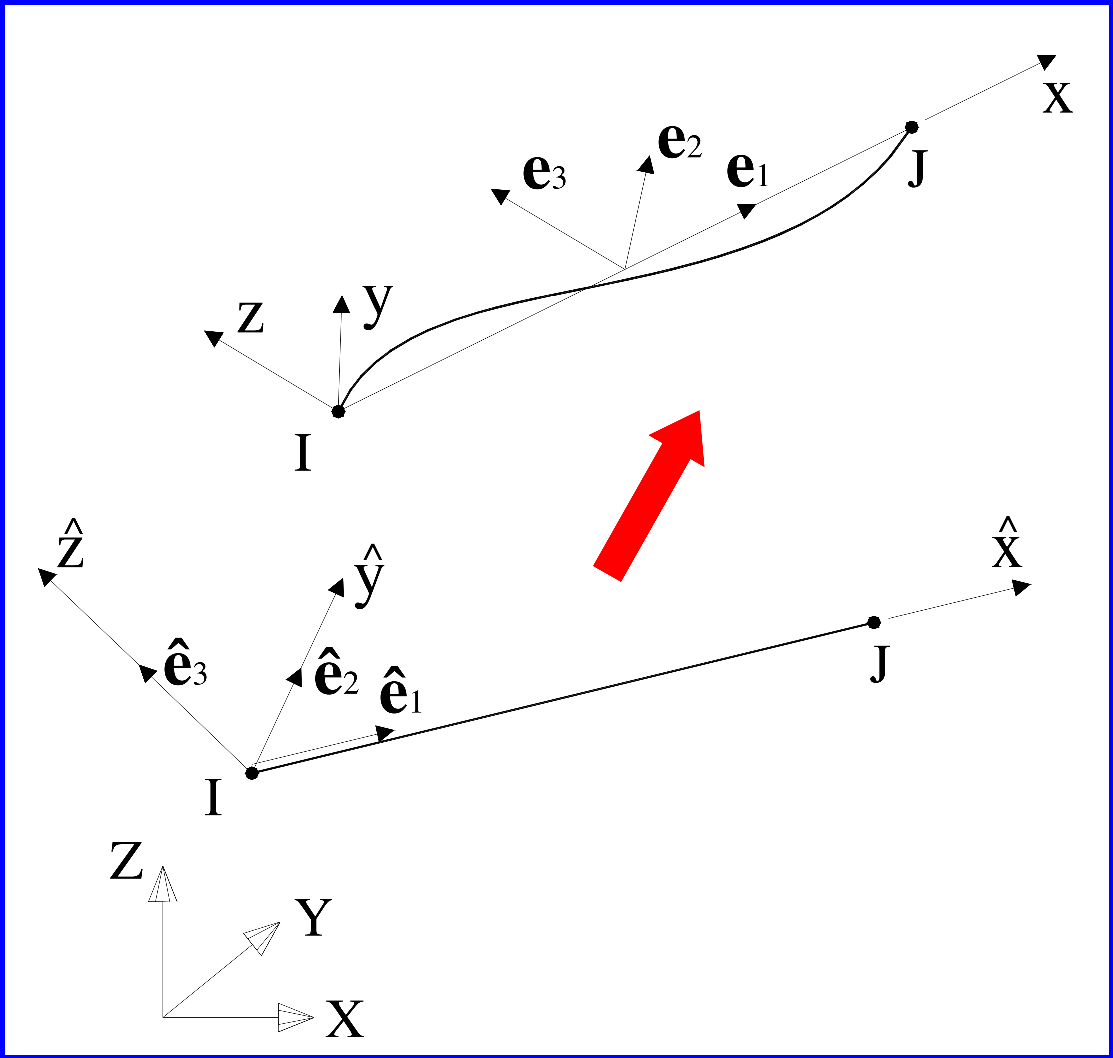
The cross section forces are obtained by a fiber approach

$$N_z = \sum_{i=1}^{n} (\sigma(\varepsilon)A)_i \qquad M_x = \sum_{i=1}^{n} (\sigma(\varepsilon)\,yA)_i \qquad M_y = -\sum_{i=1}^{n} (\sigma(\varepsilon)xA)_i$$

The generalized stiffnesses (or section tangent moduli) are the derivatives of the forces w.r.t. the strain variables and are also obtained using fiber integration.

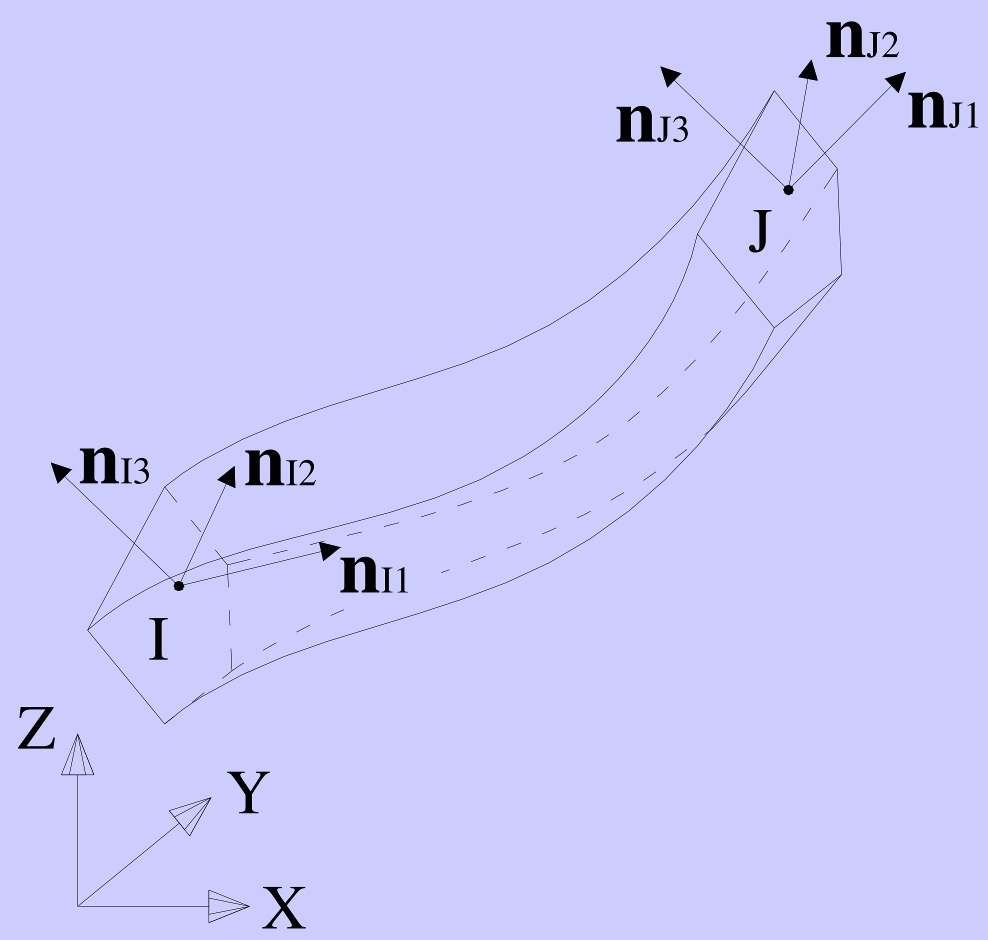
Beam-column element formulation

Initial and deformed configurations, along with element initial ($\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{e}}_2$, $\hat{\boldsymbol{e}}_3$) and current local triads ($\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$). The large displacement effect is taken into account by a corotational formulation.

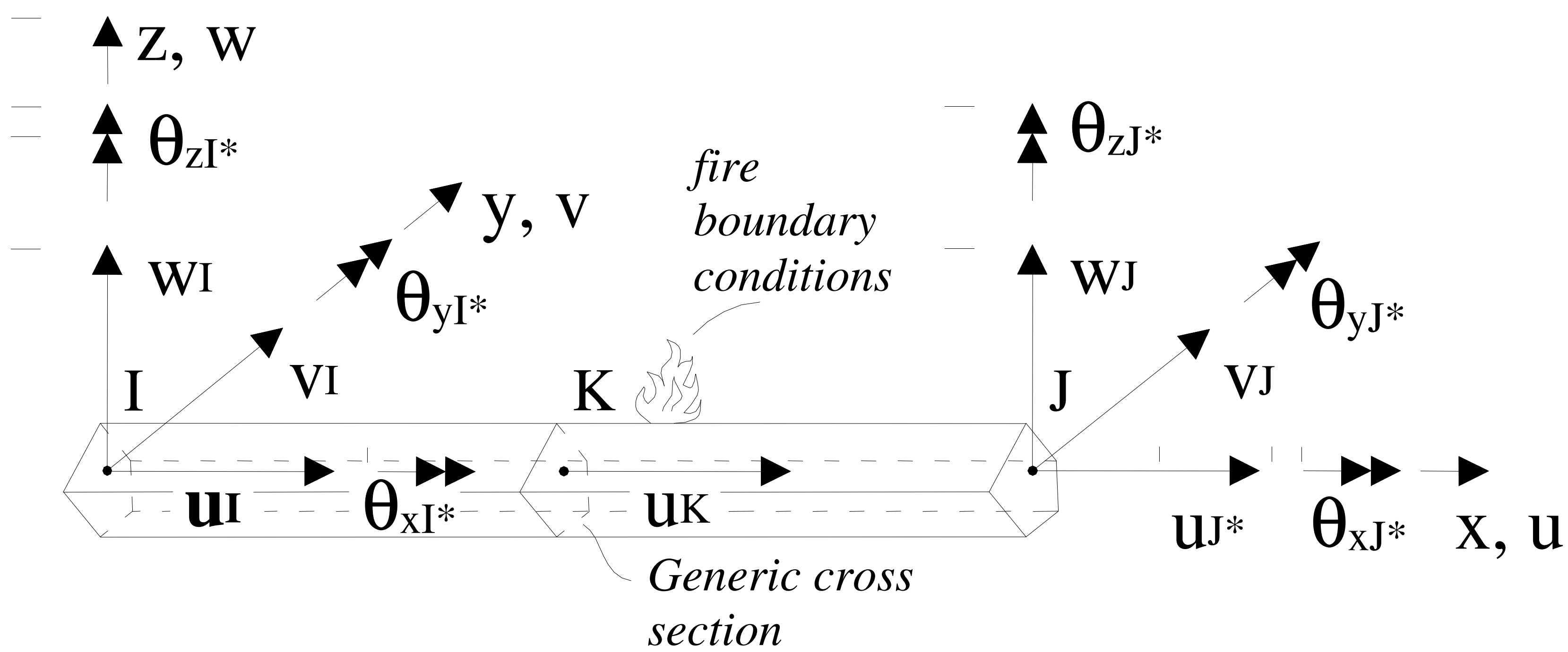


Beam-column element formulation

Deformed configurations, along with element initial and final local triads after deformation. The triad in node I rotates to n_{Ii} and the triad in node J rotates to n_{Ji}



Beam-column element formulation



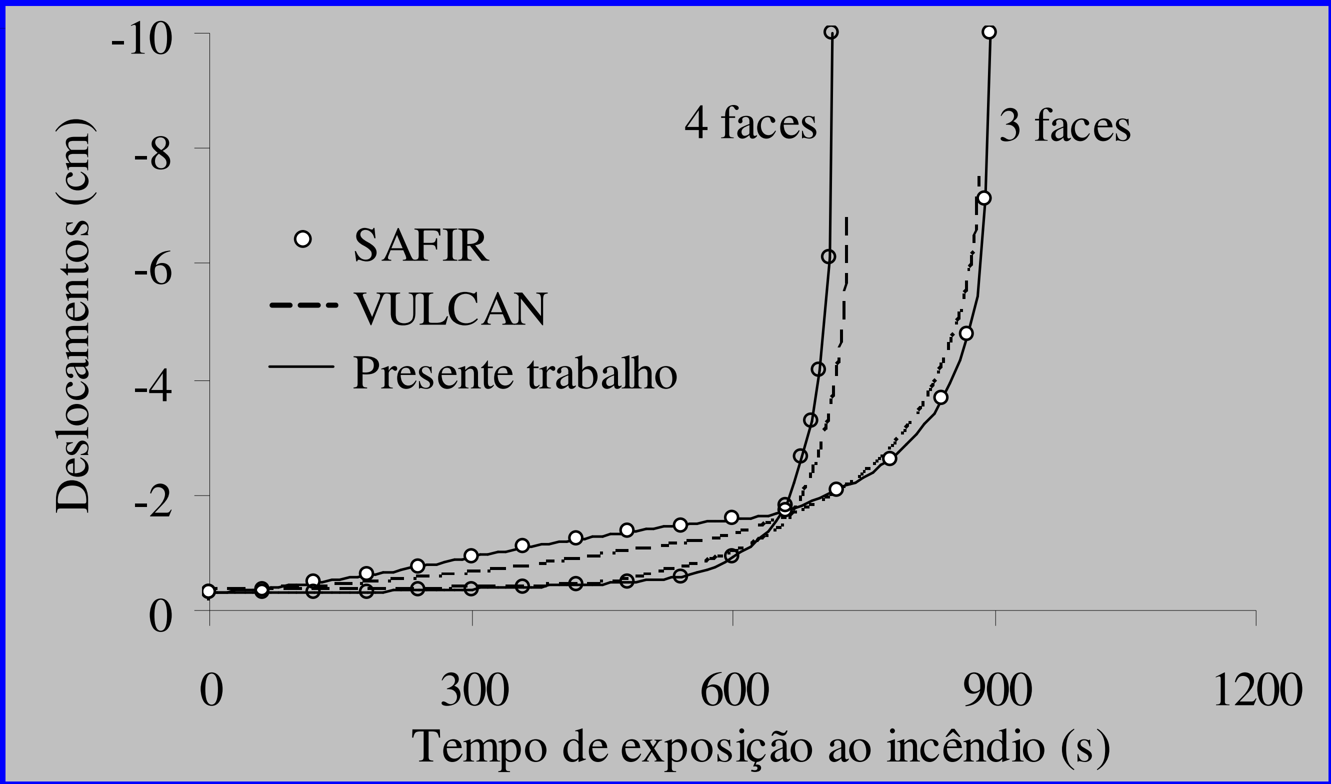
Beam-column element formulation

Example 1. Steel column

- pin-ended column subjected to standard fire on 3 and 4 faces
- section IPE 360
- at ambient temperature subjected to axial force and single curvature bending correspondent to 30% and 20% of the respective cross section plastic resistances
- results compared with SAFIR and VULCAN 10.0

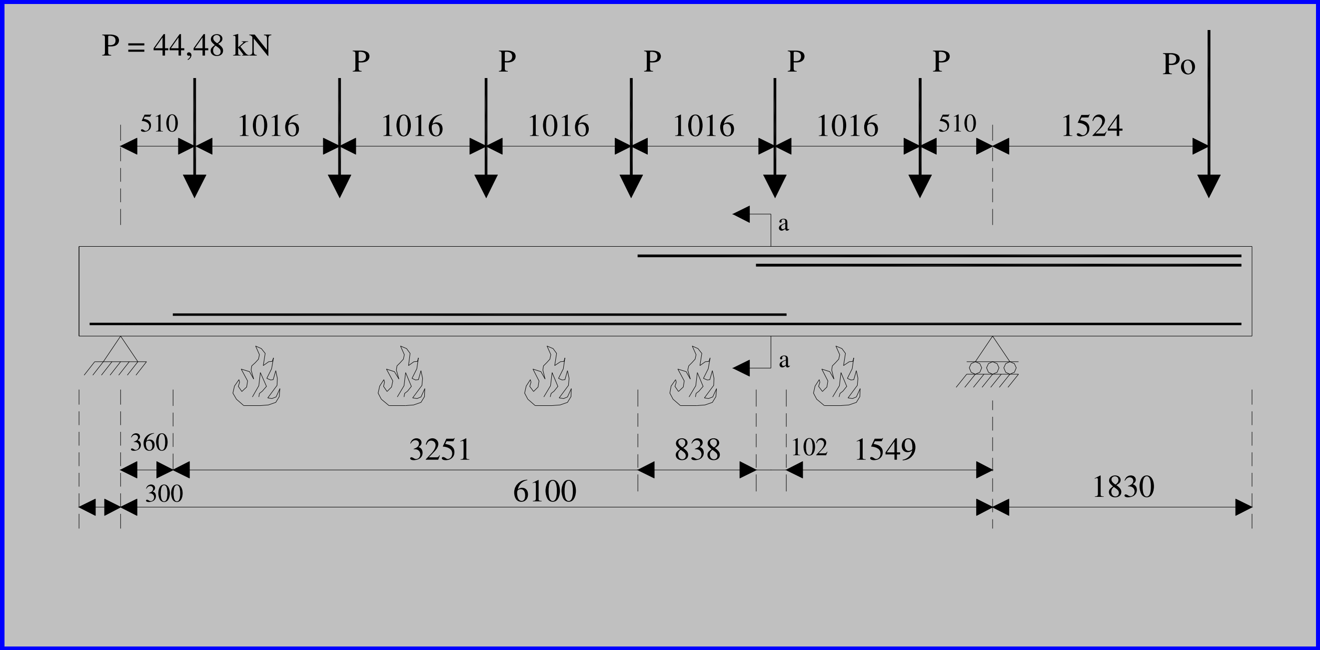
Beam-column element formulation

Results for displacements (midspan)



Beam-column element formulation

Example 2. Reinforced concrete beams tested by Ellingwood and Lin (1991) and analyzed with VULCAN por Cai et al. (2003).



Beam-column element formulation

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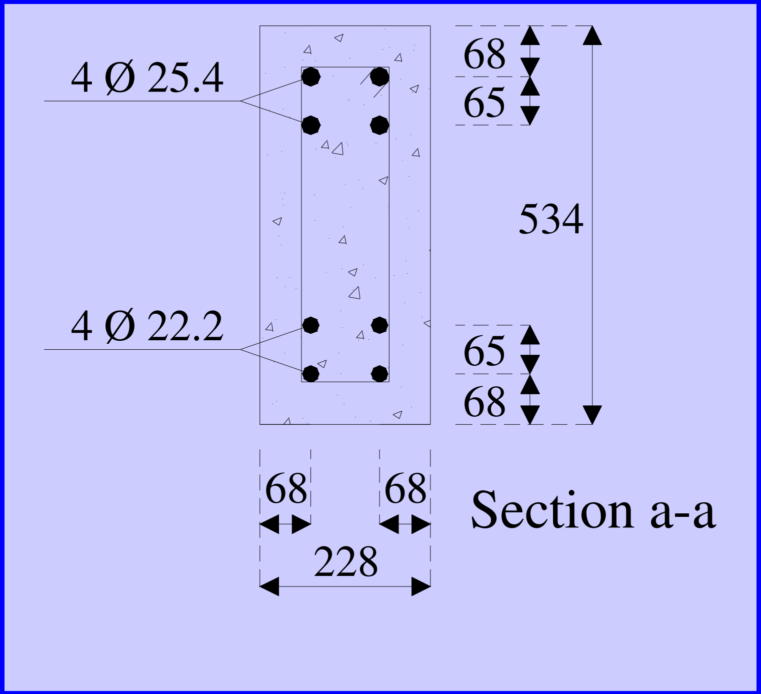


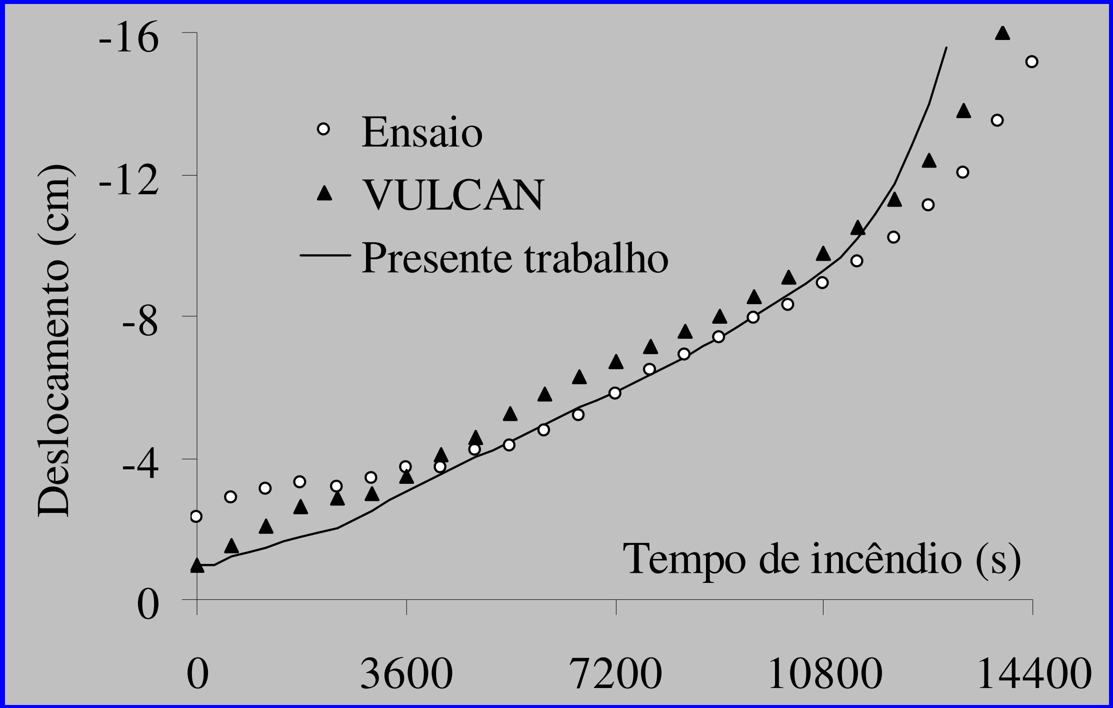
Ellingwood B., Lin T.D. (1991). Flexure and Shear Behaviour of Concrete Beams During Fires. ASCE Journal of Structural Engineering, 1176(2), 440-58.

Cai, J., Burgess, I.W., Plank, R.J. (2003). A Generalised Steel/Reinforced Concrete Beam-Column Element Model for Fire Conditions. Engineering Structures, 25(6), 817-833.

Beam-column element formulation

Example 2. Reinforced concrete beams tested by Ellingwood and Lin (1991) and analyzed with VULCAN por Cai et al. (2003).

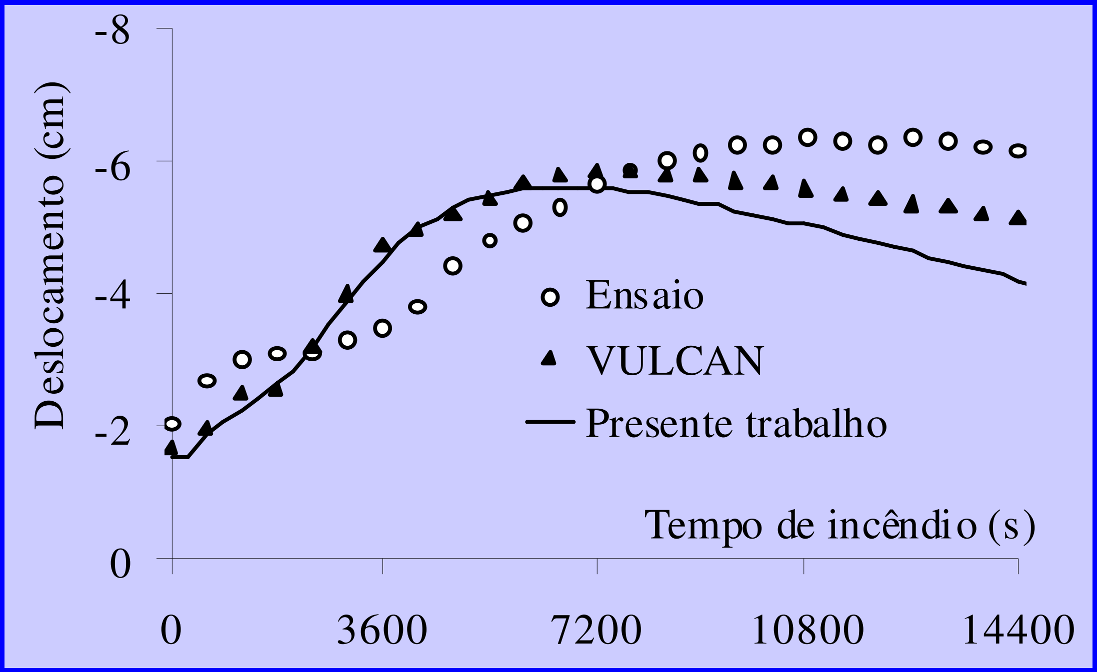
Maximum displacement results for beam 3 (test)



Beam-column element formulation

Example 2. Reinforced concrete beams tested by Ellingwood and Lin (1991) and analyzed with VULCAN por Cai et al. (2003).

Maximum displacement results for beam 6 (test)

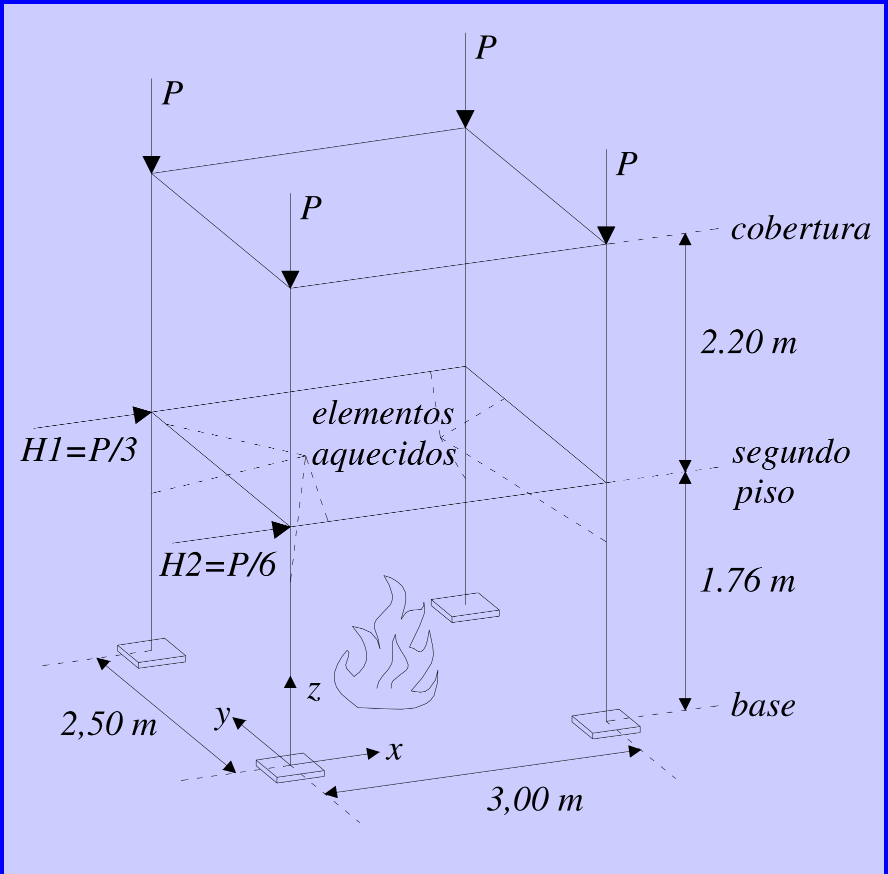


Beam-column element formulation

Example 3. 3-D steel frame analyzed by Souza Jr. and Creus (2006)

- steel has f_y=325 MPa
- cross sections H 150x150x7x10
- load P 250 kN

V. Souza Jr., G.J. Creus (2006). Simplified Elastoplastic Analysis of General Frames on Fire. Engineering Structures.



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Beam-column element formulation

Example 3. 3-D steel frame analyzed by Souza Jr. and Creus (2006)

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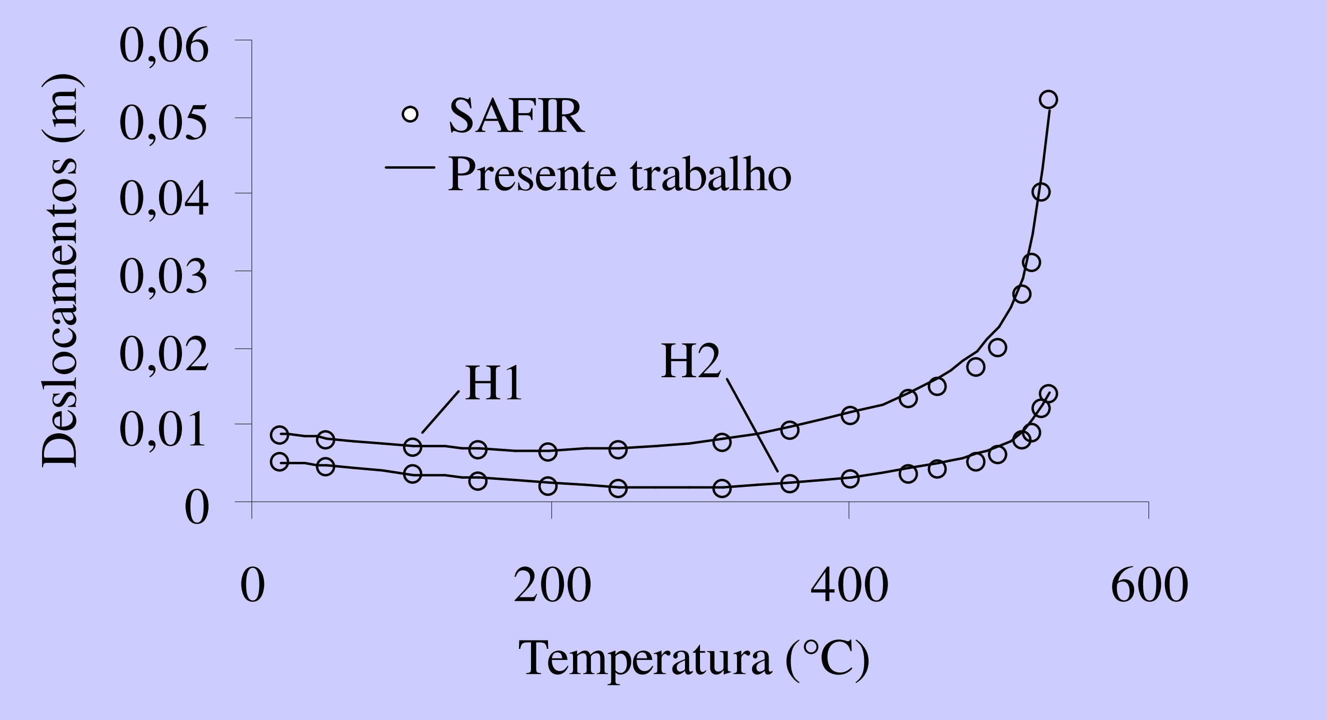


Shell element formulation

• real fires and experiments show that slabs under fire present membrane behaviour.
• several models have been developed for the numerical simulation of RC and composite slabs under fire.
• these models include Kirchhoff and Reissner-Mindlin based shell models, discrete and distributed (smeared) cracking, nonlinear geometric effects (von Karman hypothesis) and others.
• others consider composite slabs as association of beams.

Shell element formulation

• In this work a damage model was developed for the simulation of the concrete behaviour, in association with a layered approach to represent layers of concrete and reinforcement under ambient temperature and fire.
• The model assumes the existence of a compliance relation in compression and tension with smeared (distributed) crack representation.
• temperature distribution is obtained by a FD scheme through the thickness of the slab.

Shell element formulation

The model assumes a compliance relation $\boldsymbol{\varepsilon}_{12} = \mathbf{D}\boldsymbol{\sigma}_{12}$

In a local system 12 parallel to the principal strain directions.

compliance matrix is obtained from the Poisson coupling effect

$$\mathbf{D} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu}{E_0} & 0 \\ -\frac{\nu}{E_0} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Shell element formulation

The local (secant) constitutive tensor is equal to the inverse of the compliance matrix

$$\mathbf{C}_{12}^{s} = \frac{1}{1-\frac{E_1E_2}{E_0^2}\nu^2}\begin{bmatrix} E_1 & \frac{\nu E_1E_2}{E_0} & 0 \\ \frac{\nu E_1E_2}{E_0} & E_2 & 0 \\ 0 & 0 & \left(1-\frac{E_1E_2}{E_0^2}\nu^2\right)G_{12} \end{bmatrix}$$

Shell element formulation

The local (tangent) constitutive tensor is equal to

$$\mathbf{C}_{12}^{t} = \frac{\partial \boldsymbol{\sigma}_{12}}{\partial \boldsymbol{\varepsilon}_{12}} = \frac{\partial}{\partial \boldsymbol{\varepsilon}_{12}}\left(\mathbf{C}_{12}^{t}\,\boldsymbol{\varepsilon}_{12}\right) = \mathbf{C}_{12}^{s} + \frac{\partial \mathbf{C}_{12}^{s}}{\partial \boldsymbol{\varepsilon}_{12}}\boldsymbol{\varepsilon}_{12}$$

The reinforcement is considered as a layer of material with uniaxial response in the direction of the bars.

Shell element formulation

The FE geometrical nonlinear formulation employs von Karman's kinematical hypothesis

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \boldsymbol{\varepsilon}_o + z\mathbf{k} = \begin{bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{bmatrix} + z \begin{bmatrix} \dfrac{\partial \theta_y}{\partial x} \\ \dfrac{-\partial \theta_x}{\partial y} \\ \dfrac{-\partial \theta_x}{\partial x} + \dfrac{\partial \theta_y}{\partial y} \end{bmatrix}$$

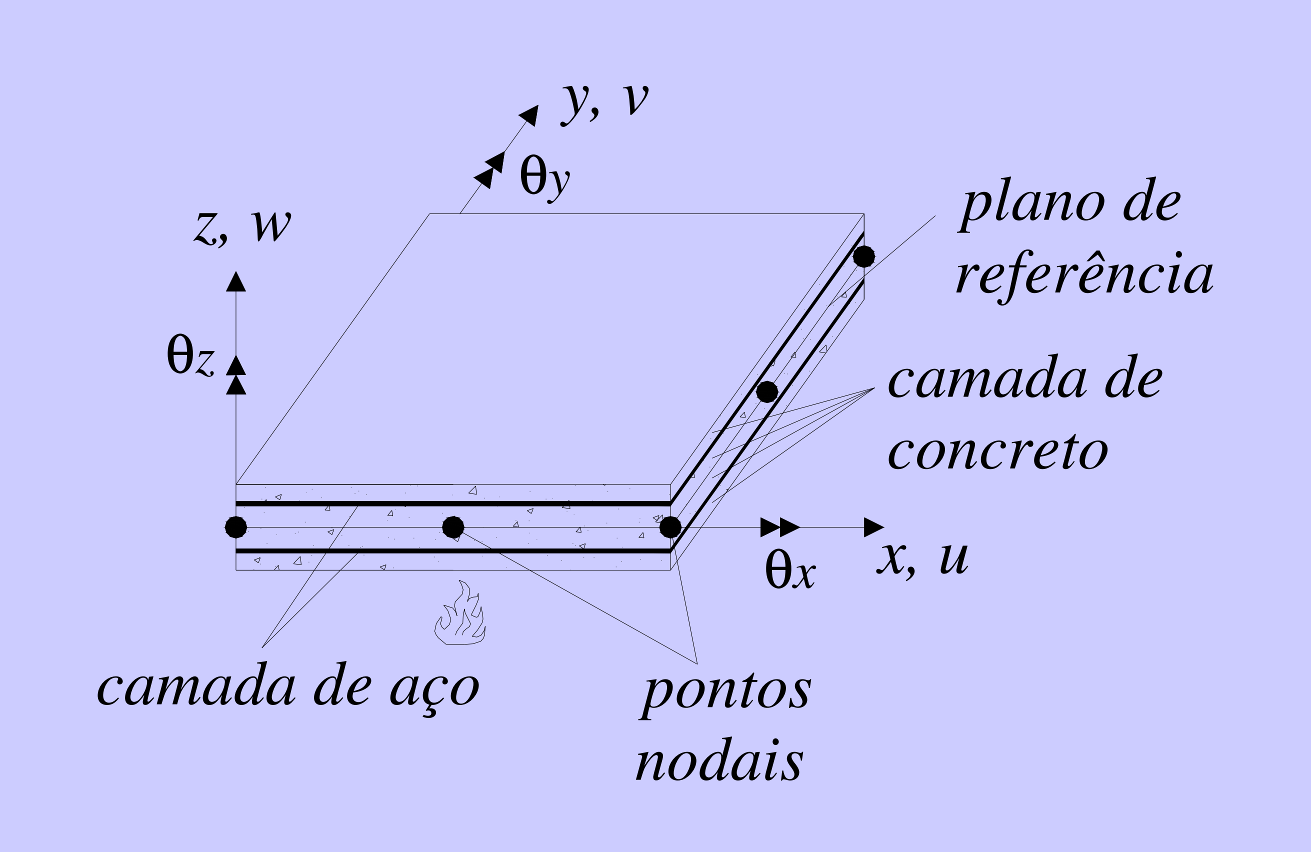
Shell element formulation

The shear strains are given by

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} \theta_y \\ -\theta_x \end{bmatrix} + \begin{bmatrix} \dfrac{\partial w}{\partial x} \\ \dfrac{\partial w}{\partial y} \end{bmatrix} = \boldsymbol{\theta} + \mathbf{s}$$

Shell element formulation

Shell finite element configuration



Shell element formulation

Using standard interpolation and strain-displacement relations, one gets the internal force vector

$$\mathbf{p}_i = \int (\mathbf{B}_\varepsilon^T \mathbf{N} + \mathbf{B}_k^T \mathbf{M} + \mathbf{B}_\gamma^T \mathbf{Q}) dA_o$$

$$\mathbf{N} = \int \boldsymbol{\sigma} dz$$

$$\mathbf{M} = \int \boldsymbol{\sigma} z dz$$

$$\mathbf{Q} = \int \alpha G \boldsymbol{\gamma}$$

Shell element formulation

Differentiation of the internal force leads to the element tangent stiffness matrix

$$\mathbf{k}_i = \int \begin{bmatrix} \mathbf{B}_\varepsilon^T \\ \mathbf{B}_k^T \\ \mathbf{B}_\gamma^T \end{bmatrix} \begin{bmatrix} \mathbf{C}_m & \mathbf{C}_{mb} & \mathbf{0} \\ \mathbf{C}_{mb}^T & \mathbf{C}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_\gamma \end{bmatrix} \begin{bmatrix} \mathbf{B}_\varepsilon \\ \mathbf{B}_k \\ \mathbf{B}_\gamma \end{bmatrix} dA_o + \int \mathbf{B}_s^T \mathbf{N}_2 \mathbf{B}_s dA_o$$

$$\mathbf{C}_m = \int \mathbf{C}^t dz \qquad \mathbf{C}_{mb} = \int \mathbf{C}^t z dz \qquad \mathbf{C}_b = \int \mathbf{C}^t z^2 dz \qquad \mathbf{C}_\gamma = \alpha G t \mathbf{I}_2$$

Shell element formulation

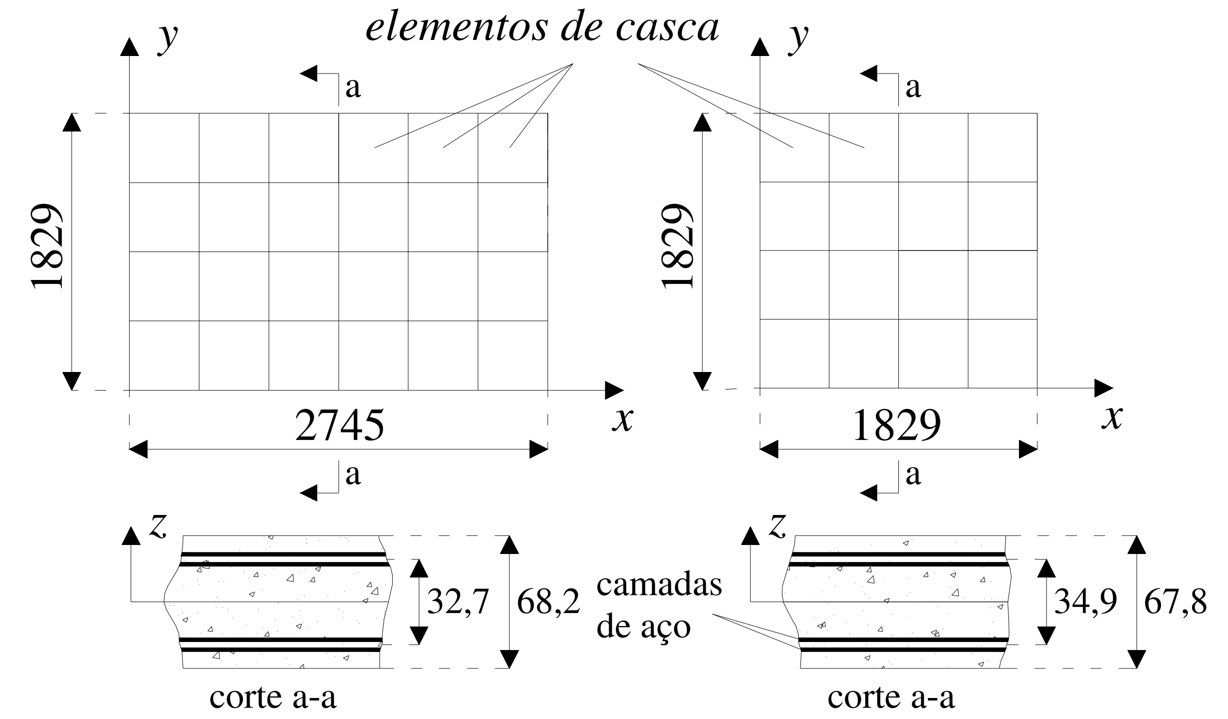
Example 1. Ambient temperature results

Ghoneim and MacGregor (1994a, 1994b) tested several RC slabs.

Huang et al. (2003b) simulated tests B1 e C1 with VULCAN

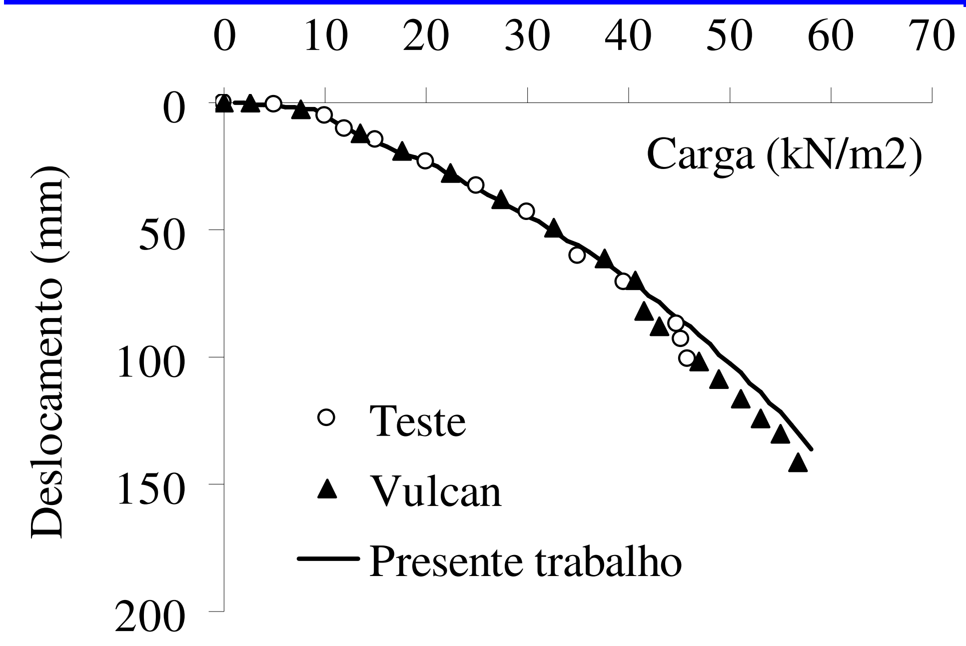
Ghoneim, M.G., MacGregor, J.G. (1994a). Tests of Reinforced Concrete Plates Under Combined Inplane and Lateral Loads. ACI Structural Journal, 91(1), 19-30.

Huang, Z. Burgess, I.W., Plank, R.J. (2003) Modelling Membrane Action of Concrete Slabs in Composite Buildings in Fire. Part I: Theoretical Development. Journal of Structural Engineering, ASCE. 2003, 129 (8), pp 1093-1102.

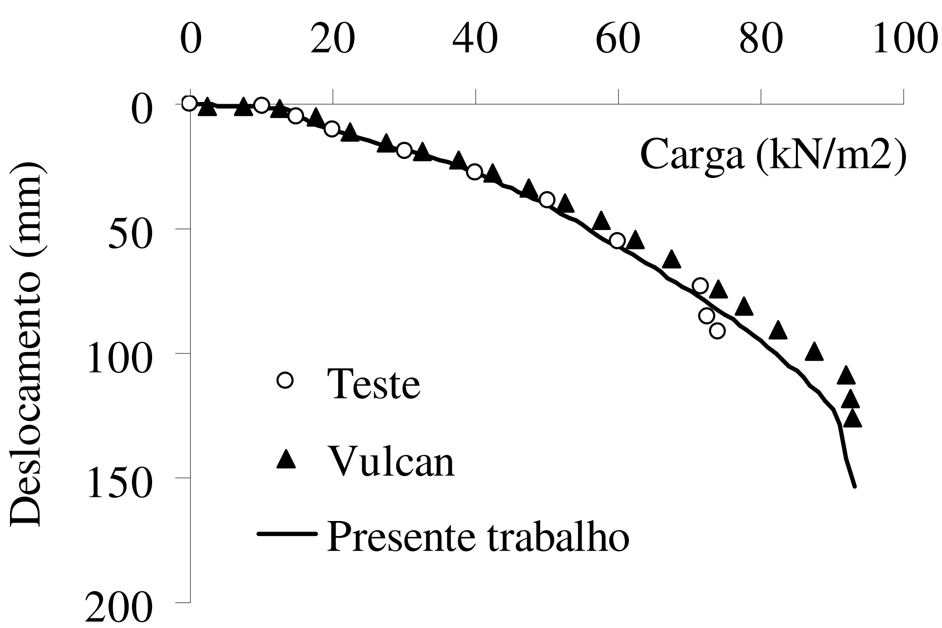


Shell element formulation

Example 1. Ambient temperature results



Test B1



Test C1

Shell element formulation

Example 2. RC slabs under fire action

Talamona and Franssen (2005) present the results of Lim and Wade (2002) of a RC slab under high temperatures

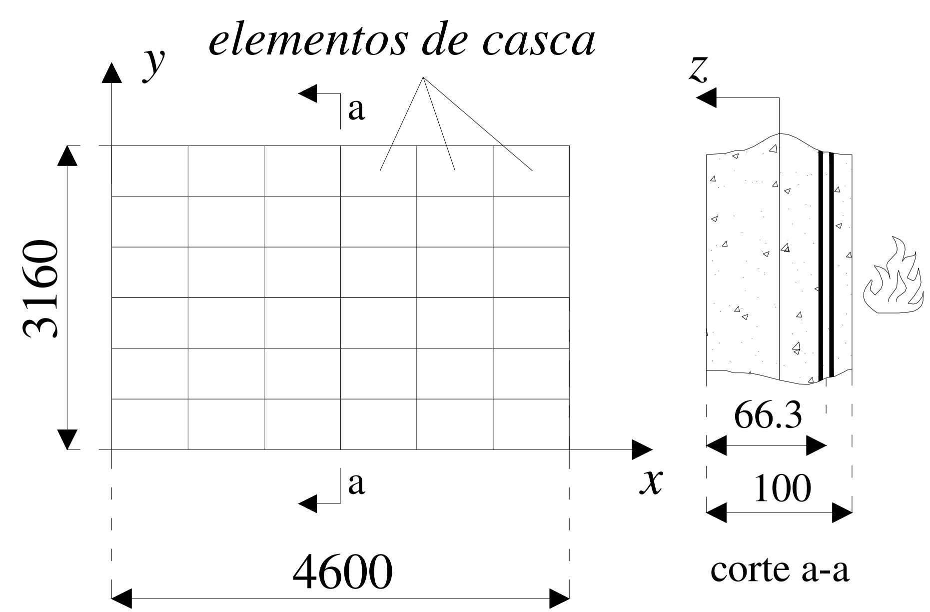
- concrete strength 36 MPa and 25 mm cover.
- reinforcement 8,7 mm every 300 mm in both directions.
- yield strength 565 MPa.
- high temperature properties from EN 1992-1-2:2004.
- limestone aggregate and upper limit for concrete thermal conductivity EN 1992-1-2:2004.
- moisture 3% of concrete weight.
- 22 layers across the slab thickness

Shell element formulation

Example 2. RC slabs under fire action

Talamona, D., Franssen, J.M. (2005). A Quadrangular Shell Finite Element for Concrete and Steel Structures Subjected to Fire. Journal of Fire Protection Engineering, 15, 237-264.

Lim, L., Wade, C. (2002). Experimental Fire Tests of Two-Way Concrete Slabs. Fire Engineering Research Report 02/12, Department of Civil Engineering, University of Canterbury, New Zealand.



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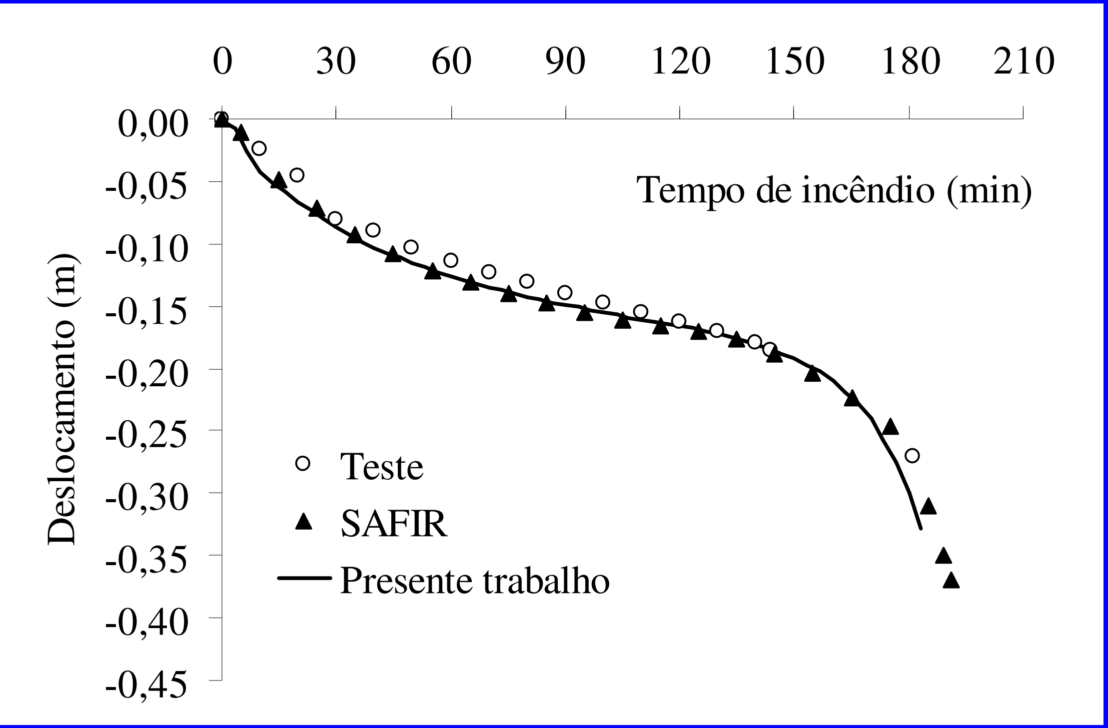
Shell element formulation

Example 2. RC slabs under fire action

Results for displacements

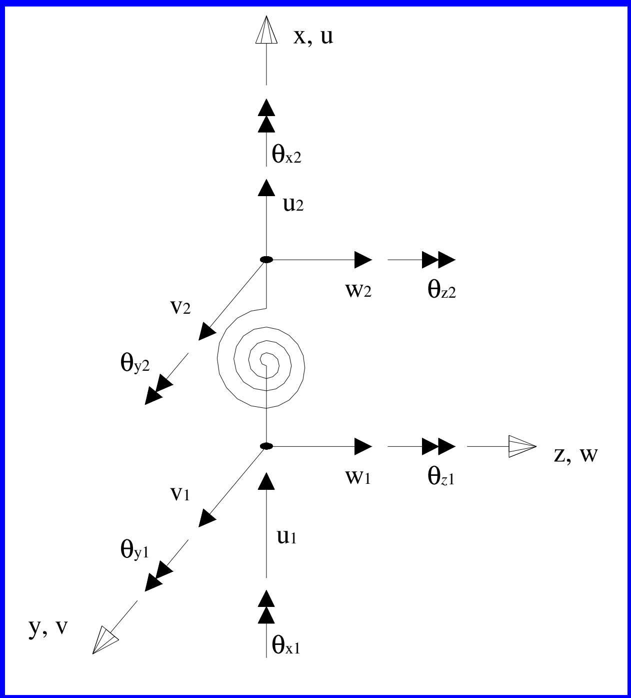


Spring element

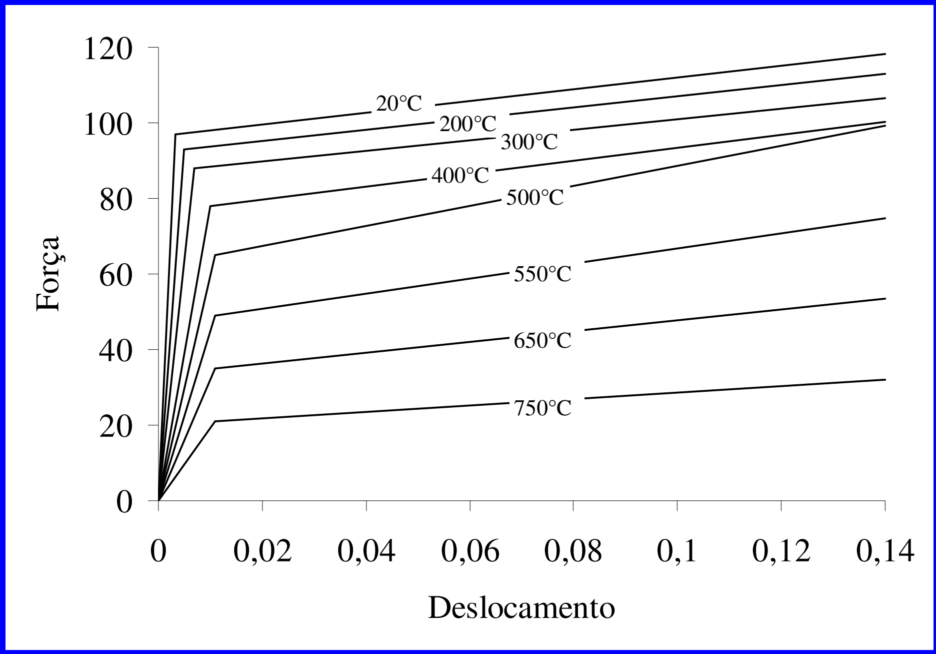
- In order to model connections under fire action a simple 6-dof zero-length spring element was implemented.
- The element may be employed to simulate semi-rigid, composite and shear connectors under fire on its own or as part of a more complex assemblage such as in component method.
- Constitutive relations are assumed to be bilinear and temperature dependence is taken into account.

Spring element

Element



Constitutive relation



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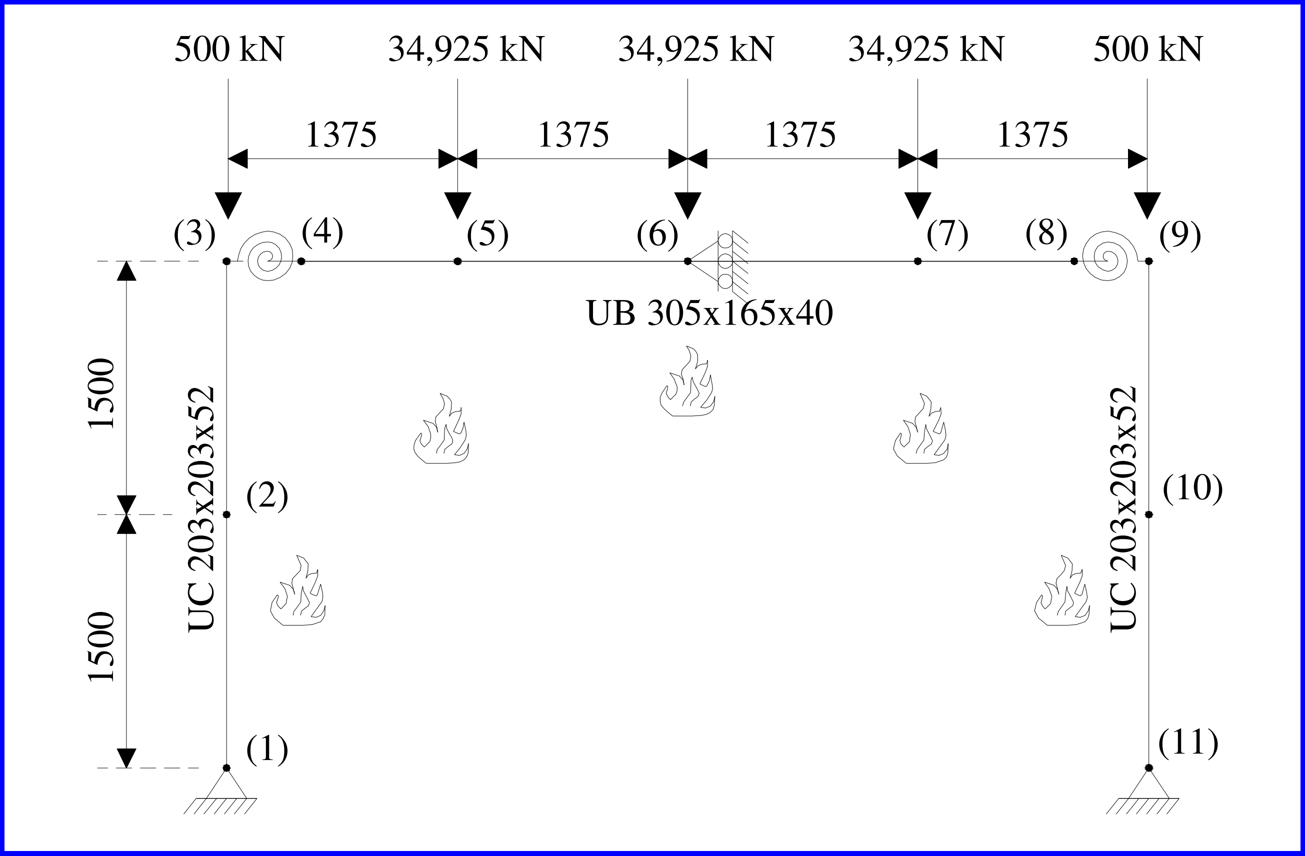
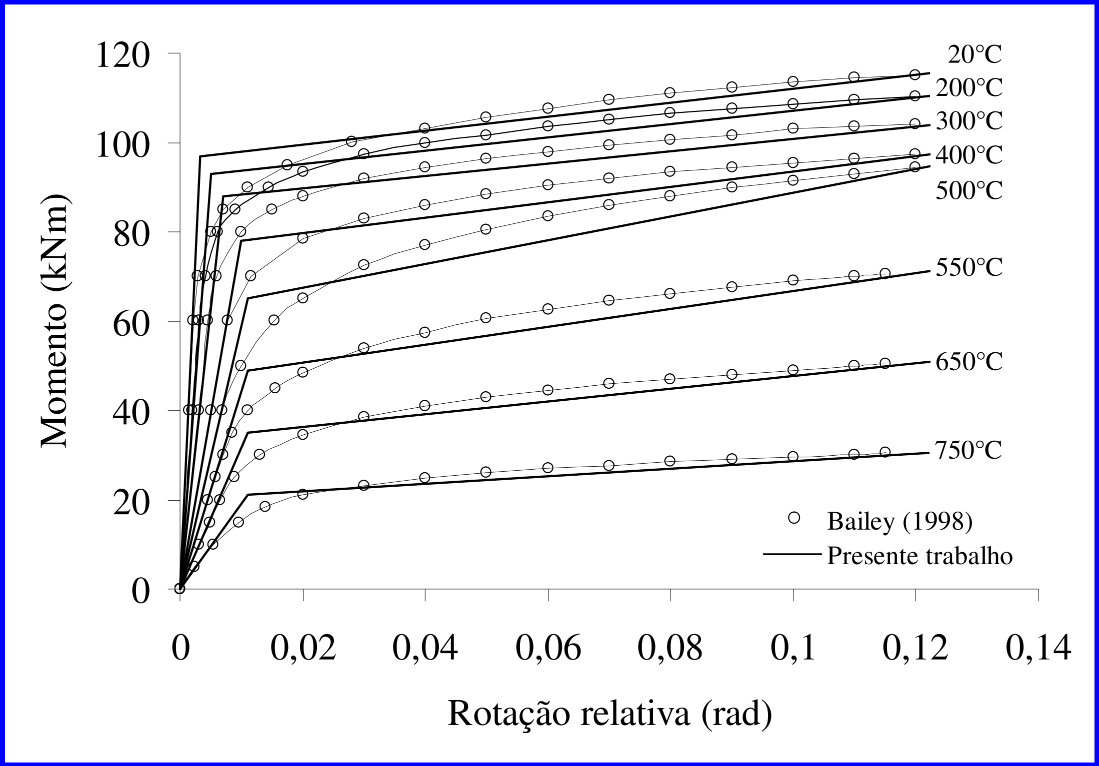
Spring element

- Semi-rigid frame analyzed by Bailey (1998)



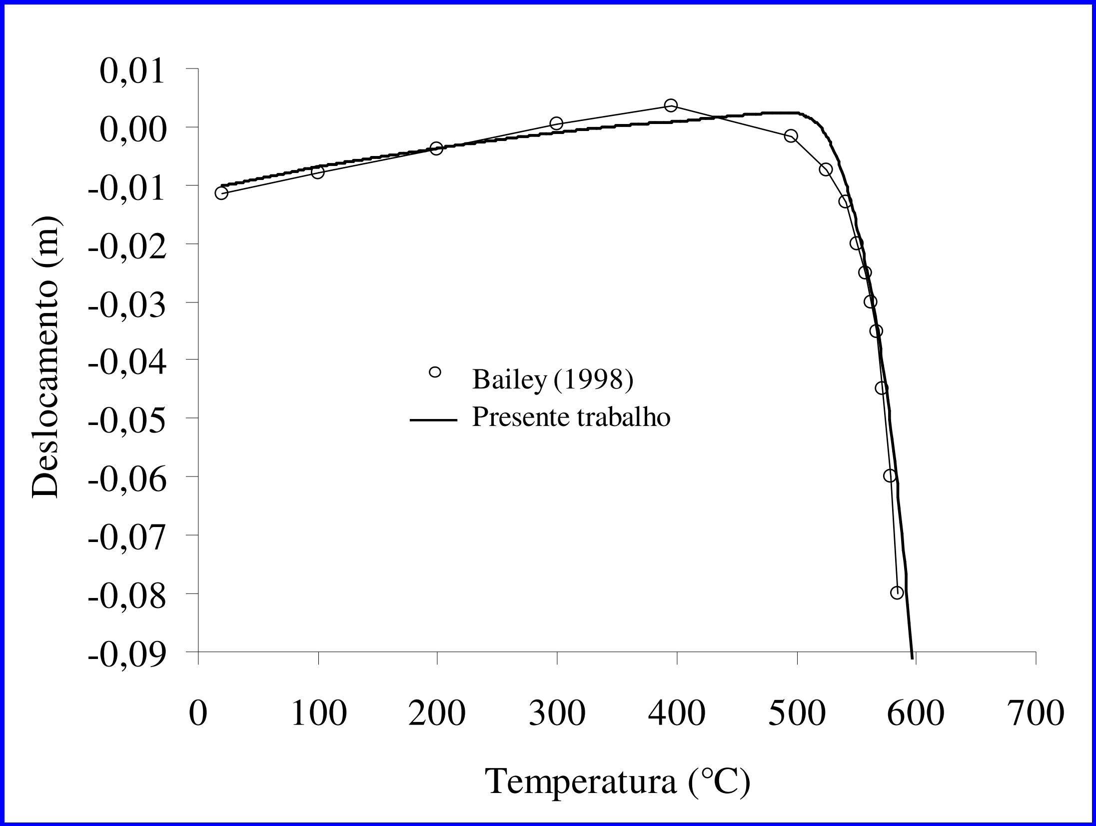
Spring element

- Approximation of moment-curvature relations by Bailey (1998)



Spring element

- Comparison of results for displacement



Summary and conclusions

- A C++ finite element analysis program was extended to the case of structural analysis of steel, reinforced concrete and composite structures under fire.
- Heat transfer analysis is coupled into the process by finite difference and finite element methods at the cross section level.
- 3D large displacement corotational inelastic beam column finite elements were implemented for the case of high temperatures.
- Reinforced concrete slabs under high temperature were modeled using a damage model accounting for temperature effects.
- A simple spring element was able to simulate connections under fire action
- Results compared very well to the most reliable softwares for fire analysis (e.g. VULCAN and SAFIR).
- The program may be extended in several directions.

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