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Introduction to nonlinear finite element modeling

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2.2. Material nonlinearities

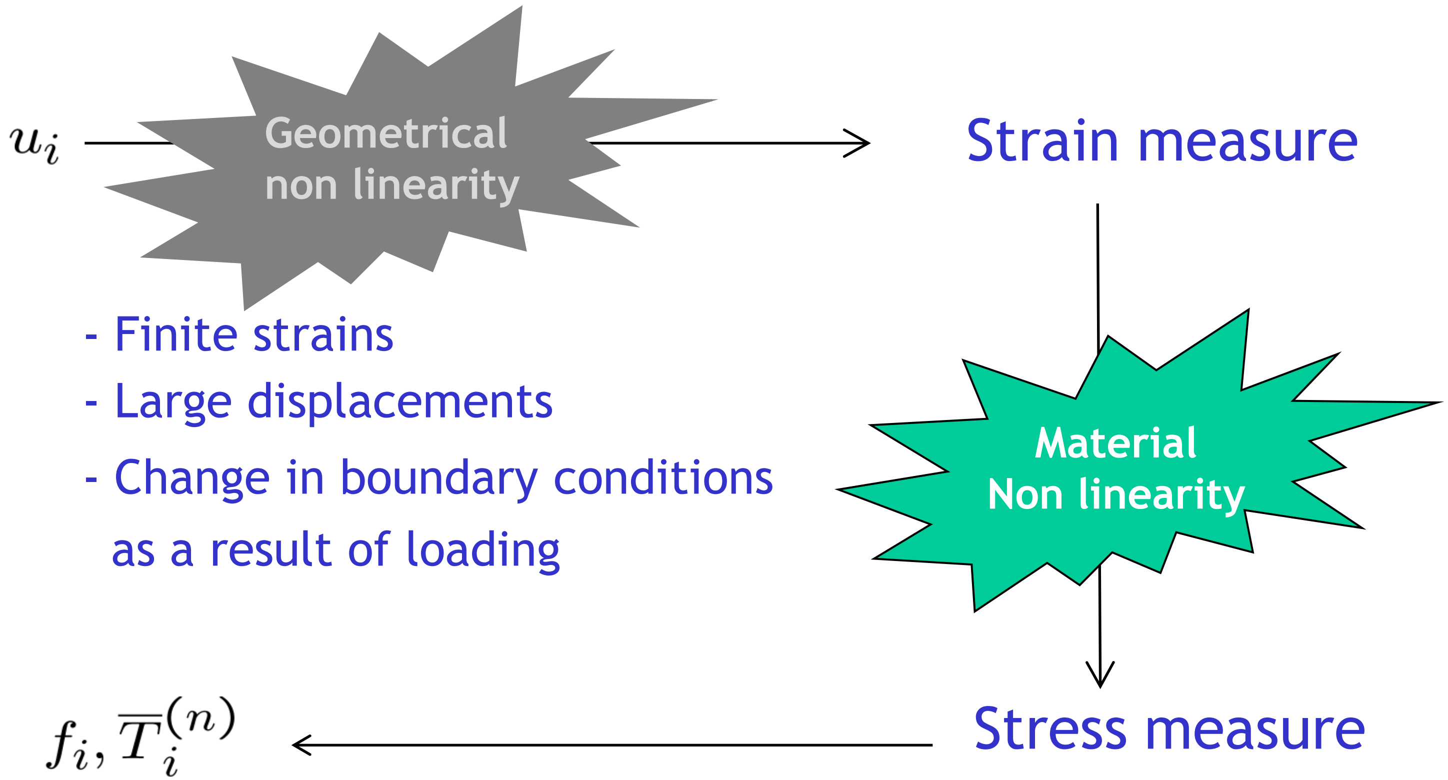
Inspired and adapted from the 'Nonlinear Modeling of Structures' course of Prof. Thierry J. Massart at the ULB

ULB

Definition

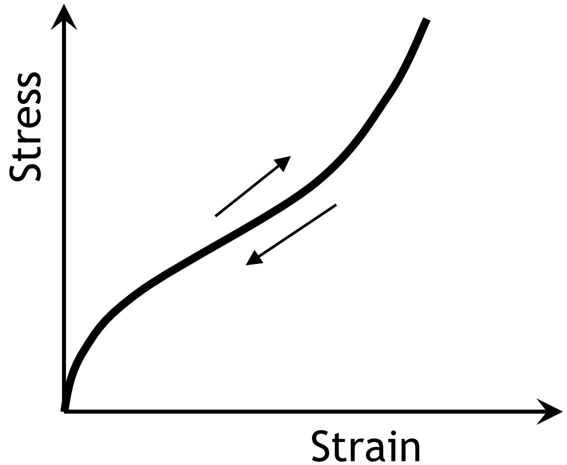
Cause of a non proportionnality between applied forces and Resulting displacements

Sources

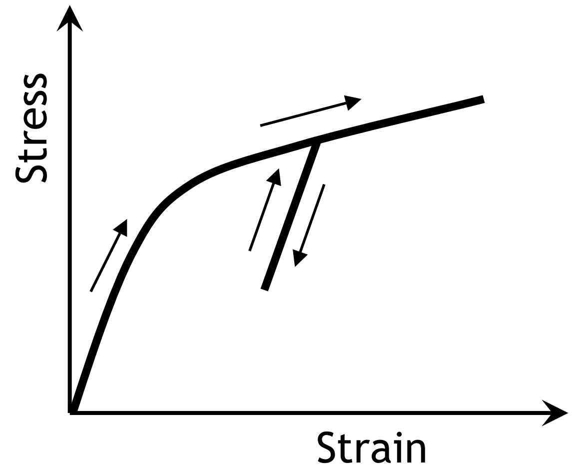


Nonlinear material behavior

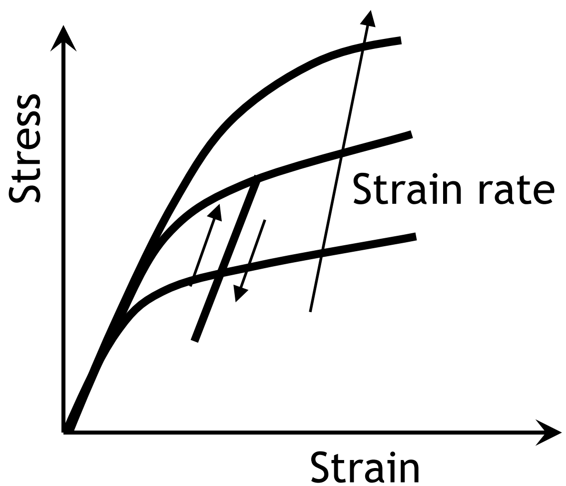
Non linear elasticity



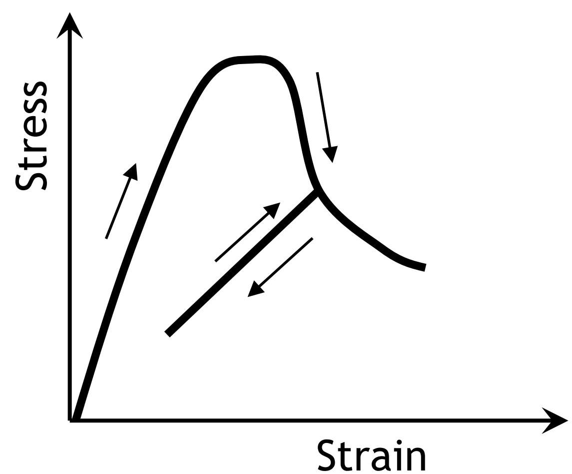
Elasto-plasticity



Elasto-visco-plasticity

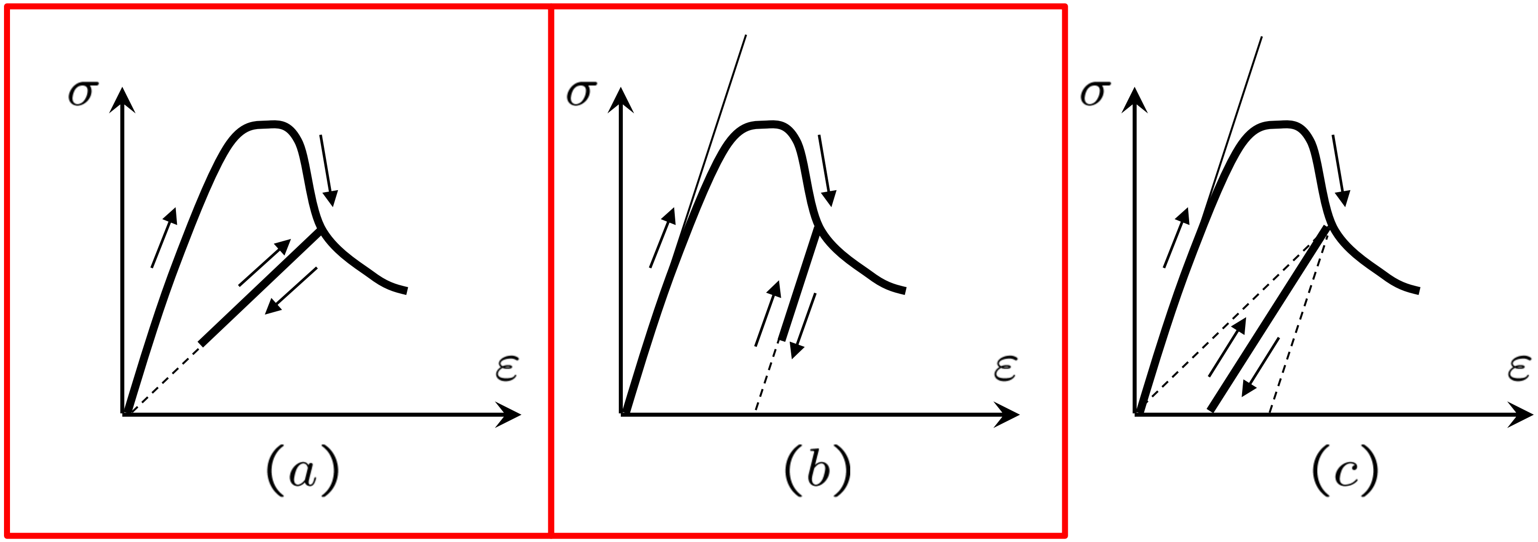


Damage (stiffness degradation)



Material non linearities - irreversibility

Classify laws in terms of types of irreversibility



(a) Irreversible degradation of stiffness

Examples: cracking, concrete under cyclic loading

(b) Irreversible (permanent) strains

Examples: plasticity, metals, soils

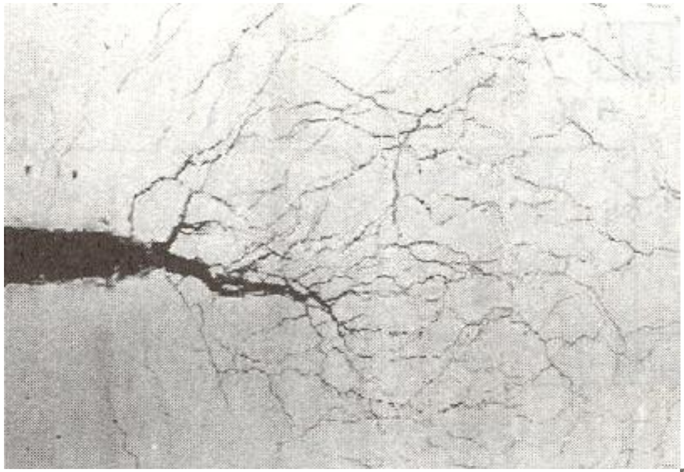
(c) Permanent strains + stiffness degradation

Damage

Plasticity

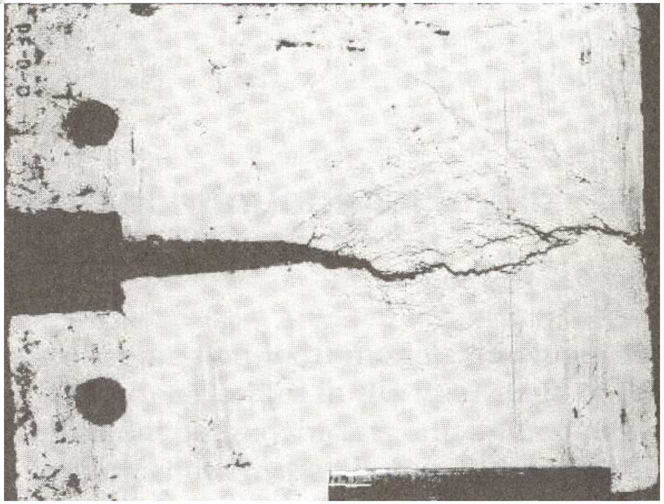
Degradation of concrete through cracking

Microcracking in concrete before failure



Ph.D. A. Simone, (TUDelft, 2003)

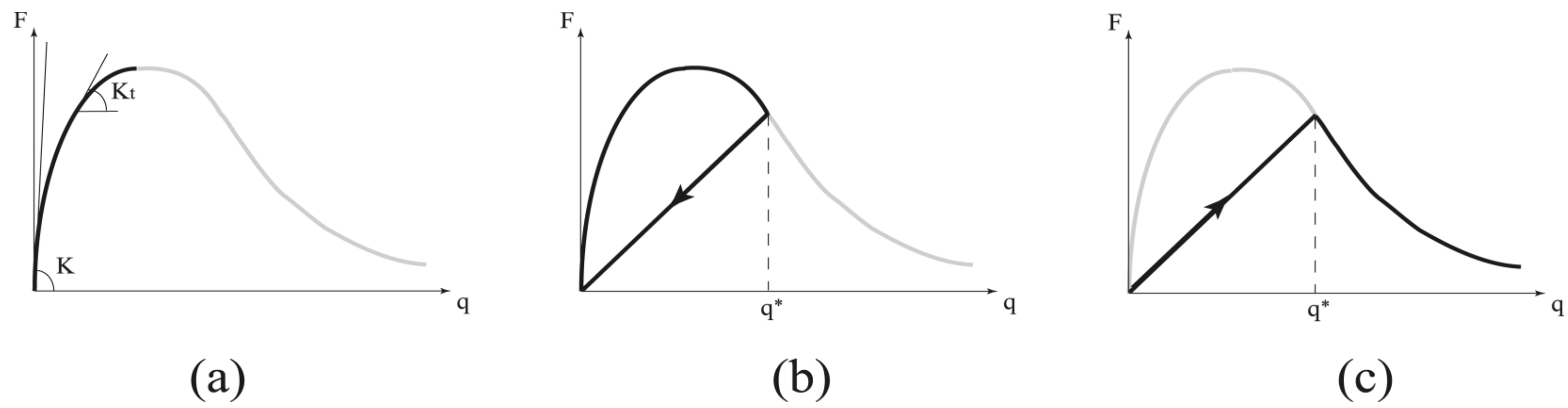
Smeared microcracking



Cracking at the macroscopic scale

A possible damage definition …

Phenomenology of damage



(a) Cracking at the microstructural scale

Smeared micro-cracking ⇒ tangent stiffness evolution

(b) Unloading with a degraded stiffness

The cracked material does not contribute to stiffness anymore

(c) Further re-loading

No NEW stiffness degradation until **q*** is reached again

→ There's a dependency of the strain history

A possible damage definition ...

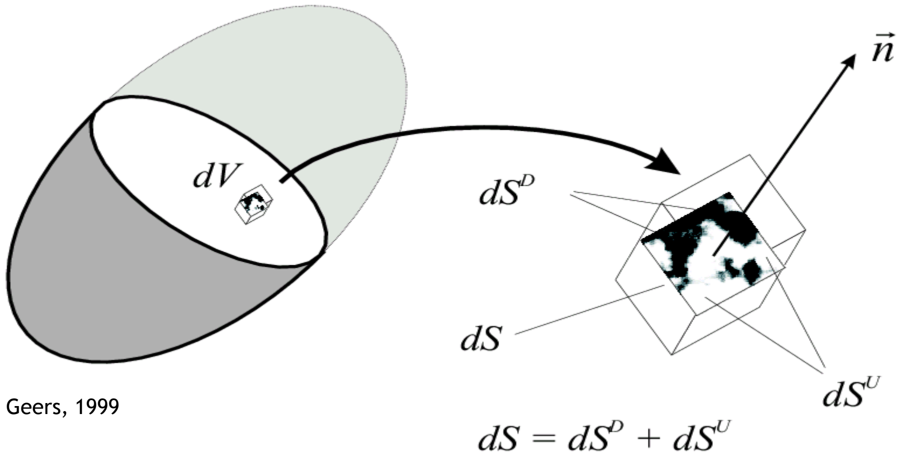
Continuum damage

A continuous variable representing the average effect of defaults

This is a variable defined in each material point of the continuum

'Micromechanical' interpretation of damage

Let's zoom on the microstructural scale & look at a section of the material



M. Geers, 1999

$dS_{\vec{n}}$ = Elementary surface

$dS_{\vec{n}}^D$ = cracked surface

$dS_{\vec{n}}^U$ = Non cracked surface

$$D_{\vec{n}} = \frac{dS_{\vec{n}}^D}{dS_{\vec{n}}} = \frac{dS_{\vec{n}} - dS_{\vec{n}}^U}{dS_{\vec{n}}}$$ = Surface density of cracks

A possible damage definition …

Continuum damage

Features of the continuous variable $D_{\vec{n}}$

- Monotonic variable reflecting irreversibility ($dS^U_{\vec{n}}$ can only decrease)
- Variable ranges between 0 and 1
- Orientation dependent quantity

Tensorial nature of damage in multiaxial cases

- The influence of microcracks is different for tangential and normal loading $\rightarrow$ vectorial damage $\vec{D}_{\vec{n}}$ associated to $\vec{n}$
- Multiaxial damage should therefore be a tensor
- Usual simplifying assumption $D_{\vec{n}} = D$

Scalar damage $\Rightarrow$ Same normal and tangential stiffness degradation!

A single continuous crack is therefore not well modelled!

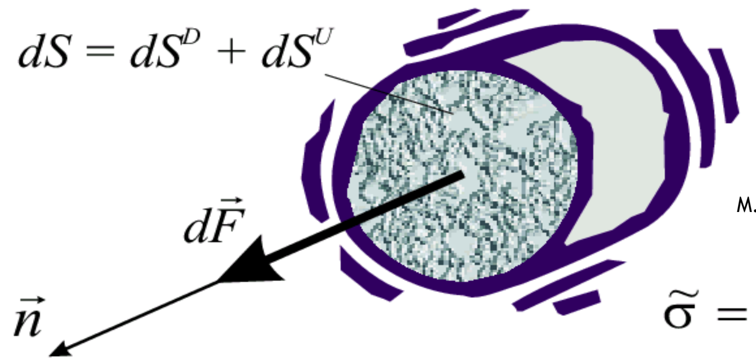
1D damaging constitutive relation

For the uniaxial case

'Nominal' stress σ = 'average' stress on the element

'Effective' stress $\tilde{\sigma}$ = stress on the resistant part of section

Relationship between nominal and effective stresses



M. Geers, 1999

$$\tilde{\sigma} = \frac{\sigma}{1-D}$$

$$\tilde{\sigma} = \frac{dF}{dS^U} = \frac{dF}{dS - dS^D} = \frac{dF}{dS(1-D)} = \frac{\sigma}{1-D}$$

1D damaging constitutive relation

For the uniaxial case

Objective = determine the relation between nominal values

(because the nominal stress is the one that appears in the equilibrium equations)

Link between nominal stress and strain

- Strain of the damaged material under σ assumed equal to the strain of the virgin material under $\tilde{\sigma}$
- The non cracked material is assumed to follow the elastic law

$$\tilde{\sigma} = E\varepsilon \quad \Longrightarrow \quad \sigma = (1 - D)E\varepsilon = \tilde{E}\varepsilon$$

$\tilde{E}$ = Effective damaged modulus = slope at unloading

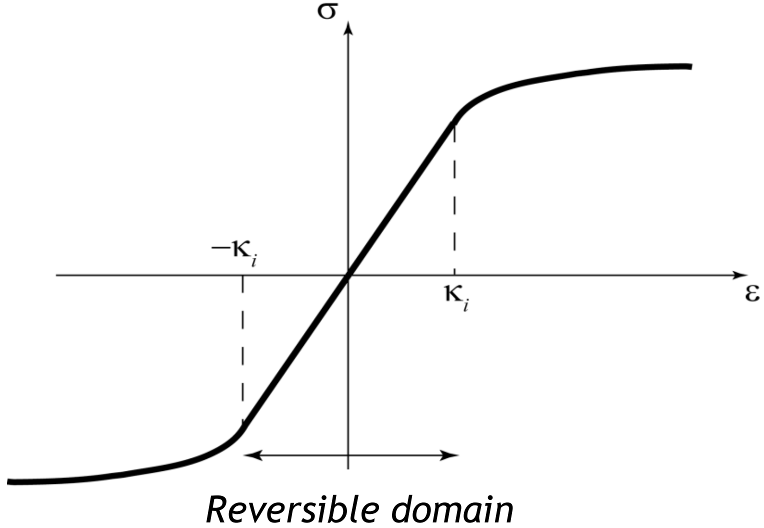
A similar development is possible with stress equivalence and effective strain

Damage evolution criterion in 1D

Initial damage criterion

Starting from a virgin state, does the variable D evolve ?

Damage is assumed to start as from a strain threshold κ_i



This graph is simplified as it assumes an identical behaviour in tension and compression

Irreversibility criterion for a state variation

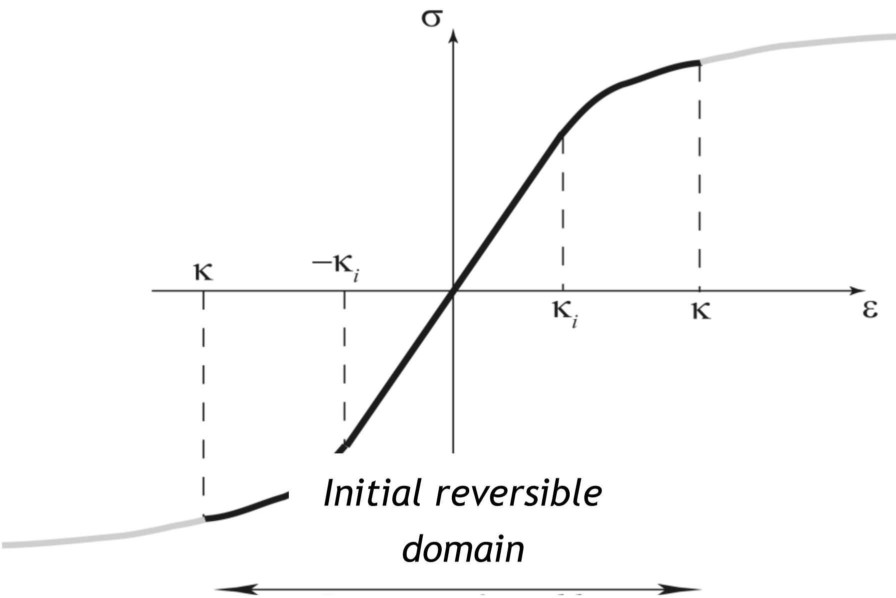
$\varepsilon \geq \kappa_i$ or $f^d = \varepsilon - \kappa_i \geq 0$ Irreversible variation

$\varepsilon < \kappa_i$ or $f^d = \varepsilon - \kappa_i < 0$ Reversible variation

The reversibility domain is expressed in the strain space

Damage criterion with evolution

Let us denote κ the most critical strain applied to the material during its history



κ is an increasing parameter measuring the accumulated irreversibilities in the material

Irreversibility criterion for a state variation (damaged case)

$\varepsilon \geq \kappa$	or	$f^d = \varepsilon - \kappa \geq 0$	Irreversible variation
$\varepsilon < \kappa$	or	$f^d = \varepsilon - \kappa < 0$	Reversible variation

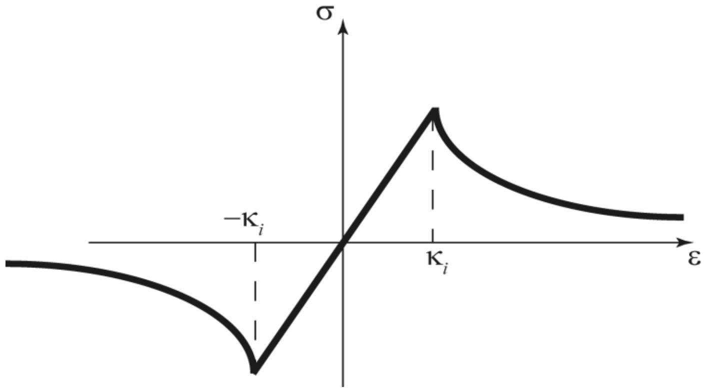
Damage criterion with evolution

If the irreversibility criterion is verified

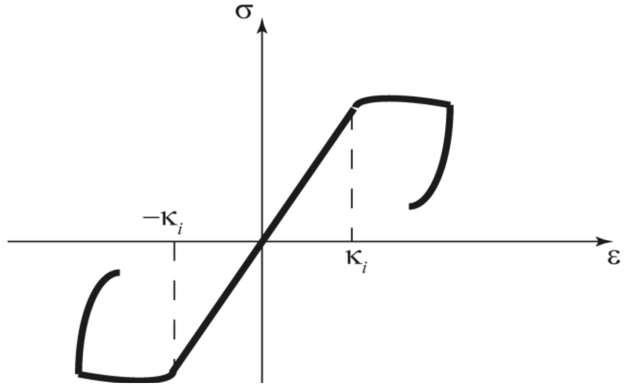
- There is a new κ for subsequent loading
- A new value of D has to be calculated

Variation of the reversibility domains in the strain space

- Its growth has to be monotonically increasing in terms of the deformation
- Its should be controlled by κ which is a monotonically increasing parameter
- This domain can only grow (and not retract) on the ε axis



Admissible



Non admissible

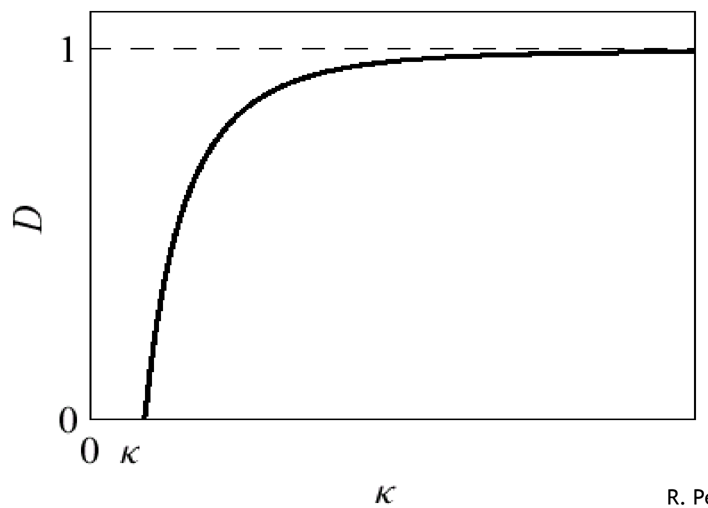
Damage evolution law

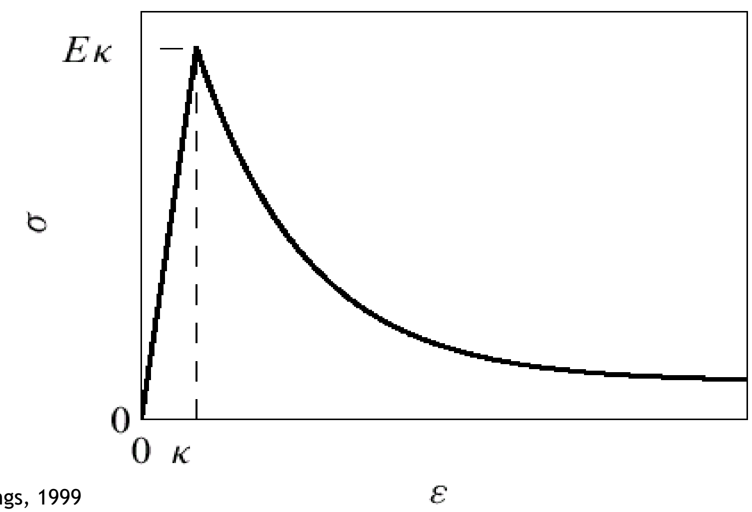
Damage should be a function of the most critical state experienced by the material: $D = D(\kappa)$

Choice of $D = D(\kappa)$ $\rightarrow$ rules the energy dissipated by the irreversible process

Choice of $D = D(\kappa)$ $\rightarrow$ ductility/brittleness of the material

Example: exponential evolution for quasi-brittle materials





R. Peerlings, 1999

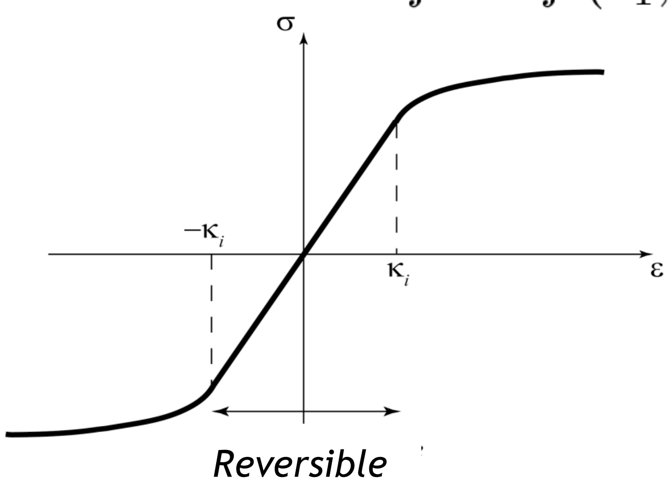
Multiaxial initial damage criterion

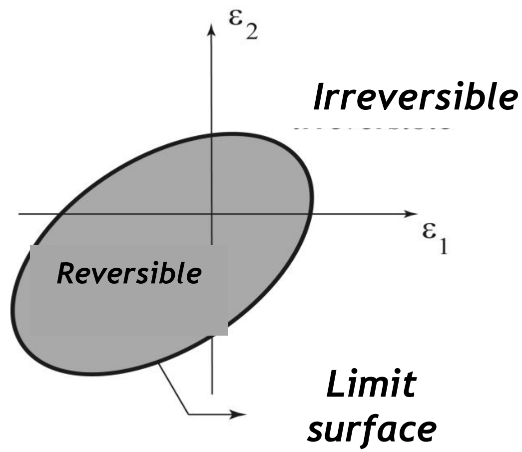
Strain state defined by principal values $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$

Limited domain of reversible states $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ in the current state

The boundary of the reversible domain is described by f^d

$$f^d = f(\varepsilon_1, \varepsilon_2, \varepsilon_3) - \kappa_i = 0$$





f^d is expressed as a function of the invariants of a_{ij} for isotropic materials (i.e. as a function of the principal values)

Multiaxial damage evolution criterion

Multiaxial initial damage criterion

Shape of the criterion

- Reflects the influence of the different components of a_{ij} on damage (some materials may be more sensible to shear for instance)
- Reflects the influence of the loading type (some materials are more sensible to tension than to compression)

Usual form for scalar damage formulations

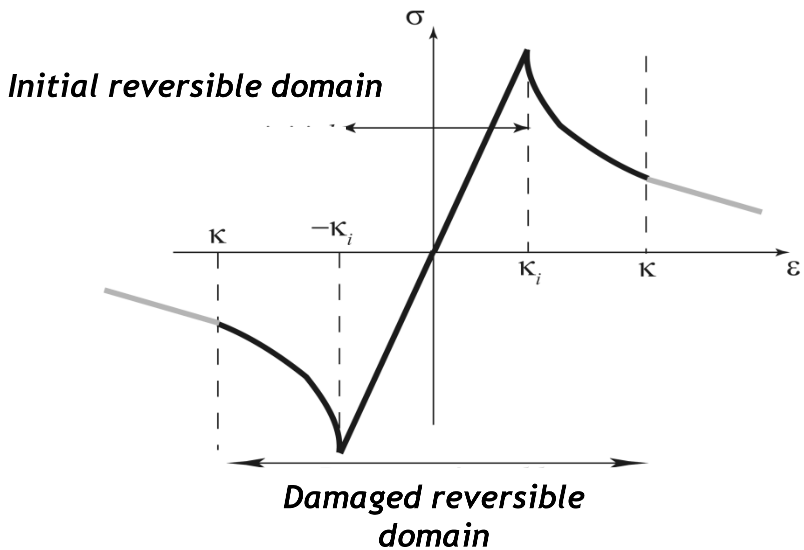
$$f^d(a_{ij}, \kappa) = \varepsilon_{eq}(a_{ij}) - \kappa$$

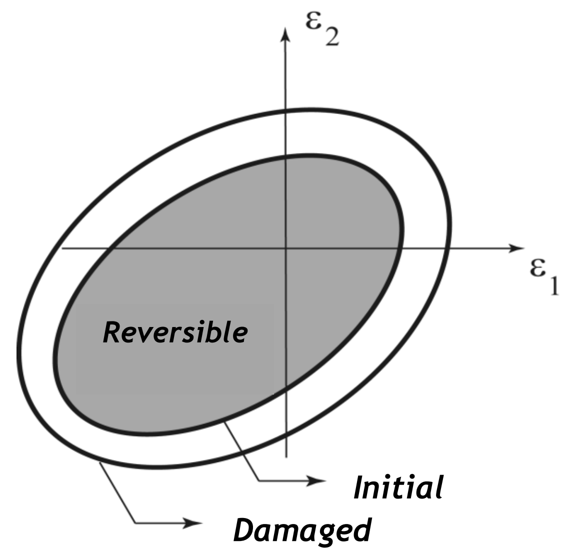
- ε_{eq} is an equivalent deformation measuring the criticality of the strain state
- The shape of the function $\varepsilon_{eq}(a_{ij})$ determines the influence of the components
- κ is in that case the most critical equivalent strain experienced

Multiaxial criterion evolution

Expansion of the reversible domain in the strain space

$$f^d = f\left(\varepsilon_1, \varepsilon_2, \varepsilon_3\right) - \kappa = 0$$



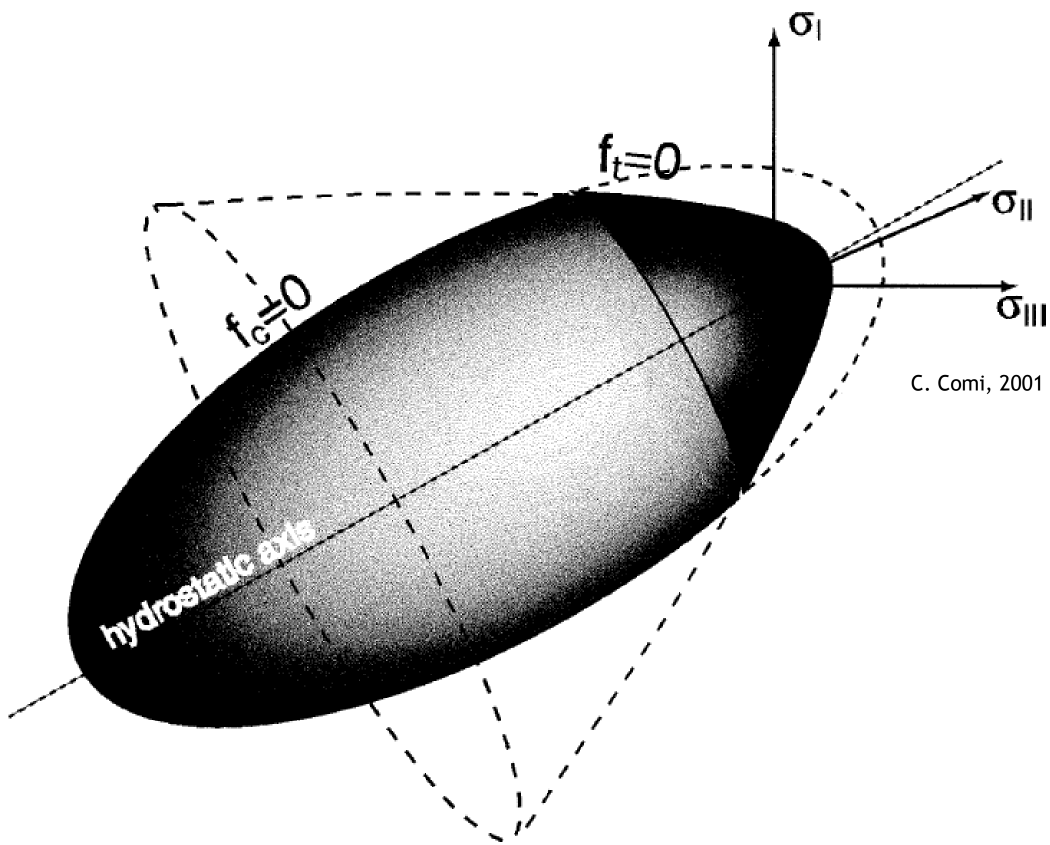


The reversible domain expands in the strain space

The domain of possible stress states may contract

Multiaxial damage evolution criterion

Criterion for concrete (3D)



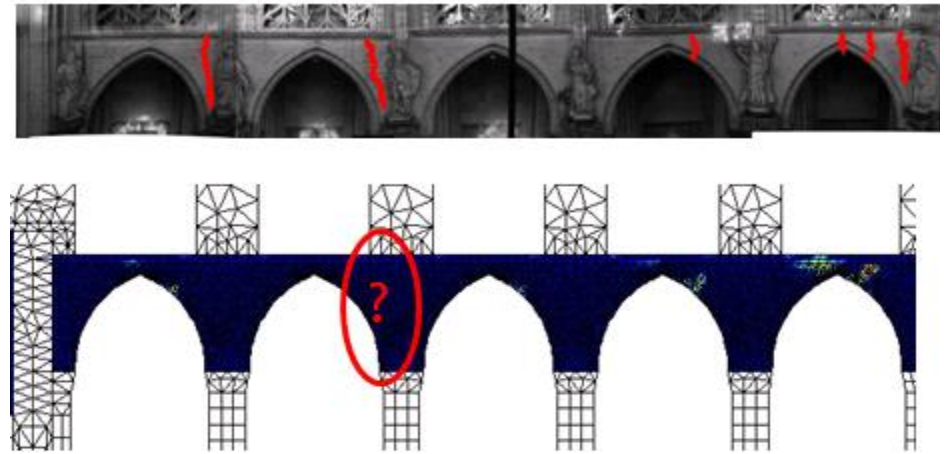
C. Comi, 2001

Expressed in stress space ⟶ to translate in strains

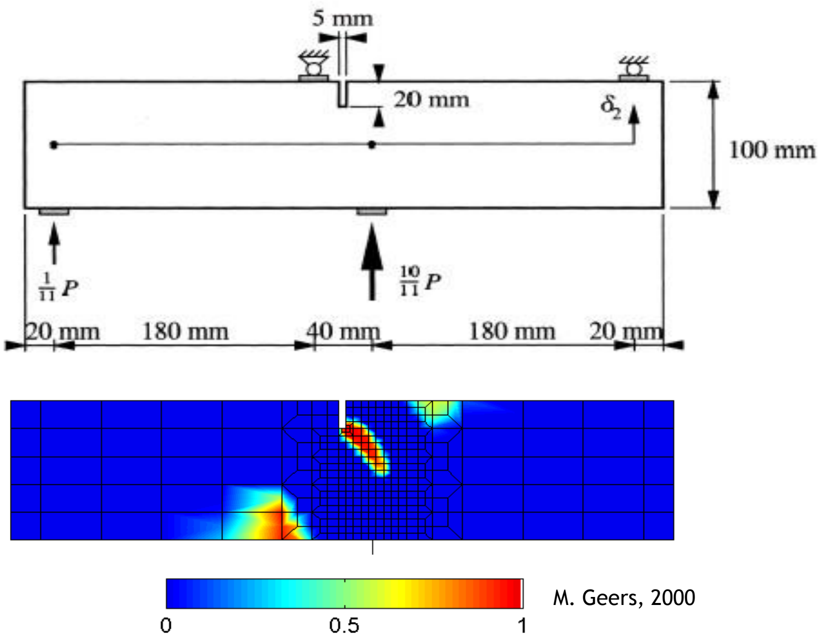
Isotropic criterion (expressed in principal stresses)

Damage mechanics - applications

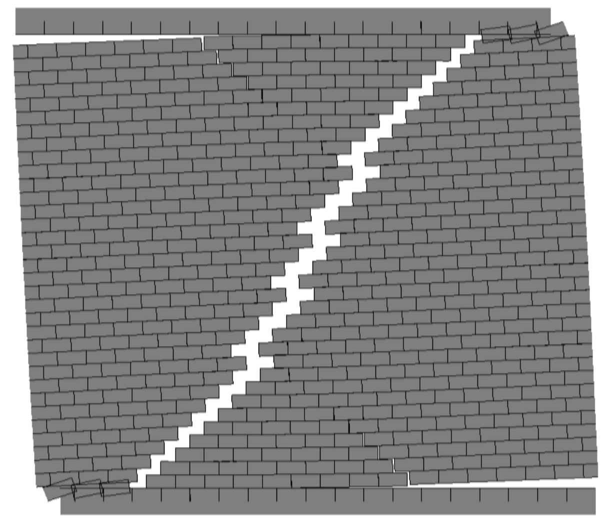
J.G.M. Wood, 2007



M. Provost



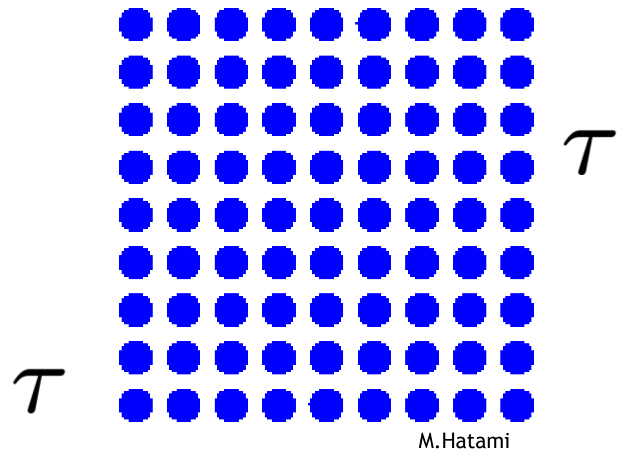
M. Geers, 2000

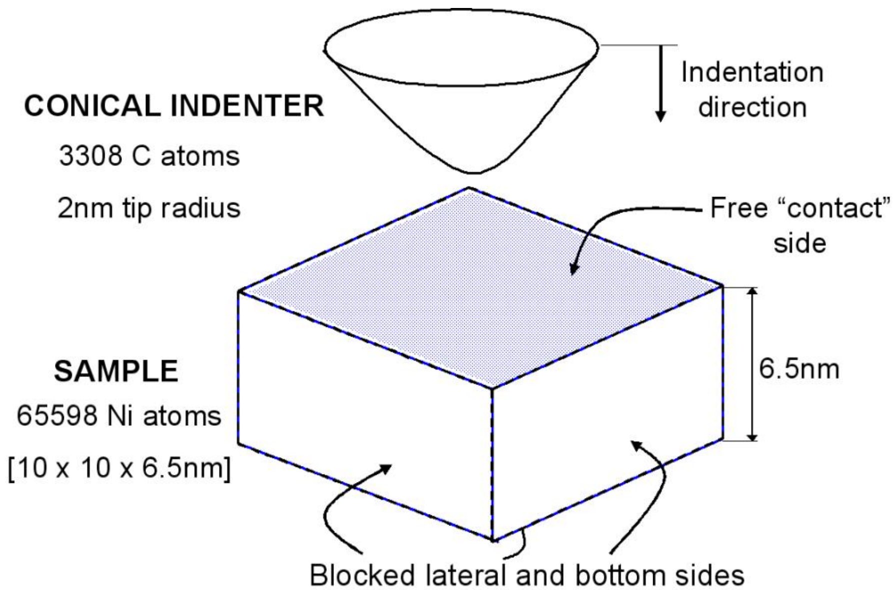


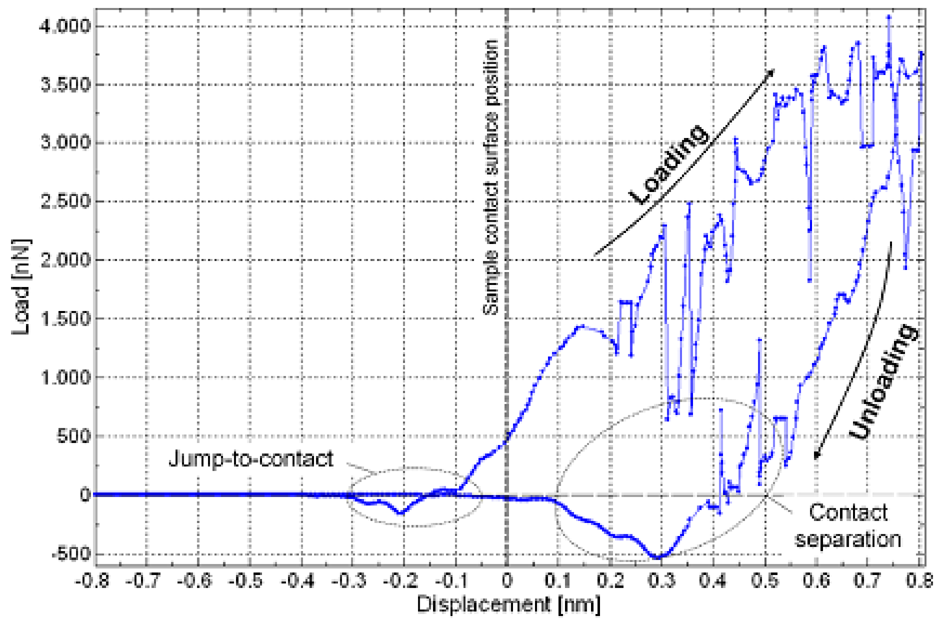
B. Mercatoris

magn = 20

Plasticity - Micromechanics







Plasticity - principles

Plasticity characteristics

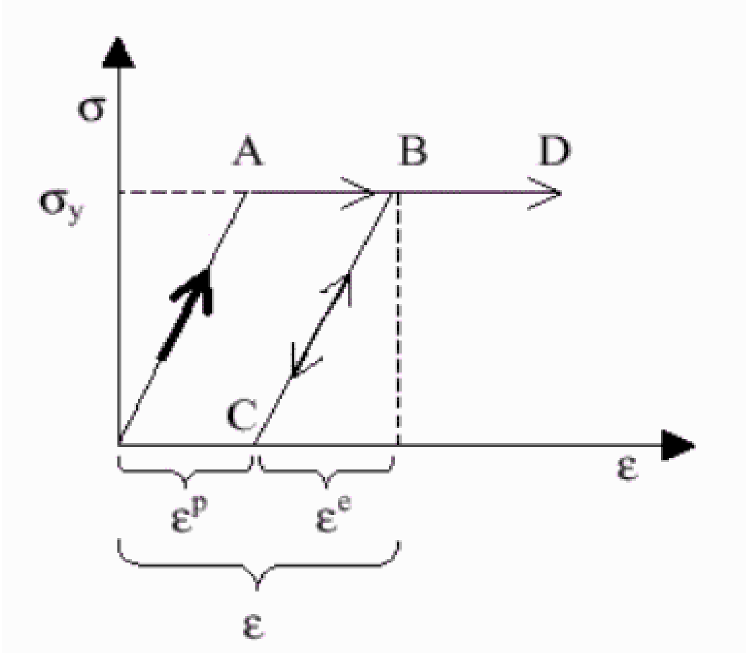
Micromechanical origins of plasticity

Perfect plasticity: glide of cristallographic planes under constant stress

Irreversibility manifests itself through permanent strains

Reversible stress states (without permanent strains) are limited by the stress level σ_y (states $\sigma > \sigma_y$ are impossible with permanent strains unchanged)

The decision of the reversibility of a state change can be made on the stress

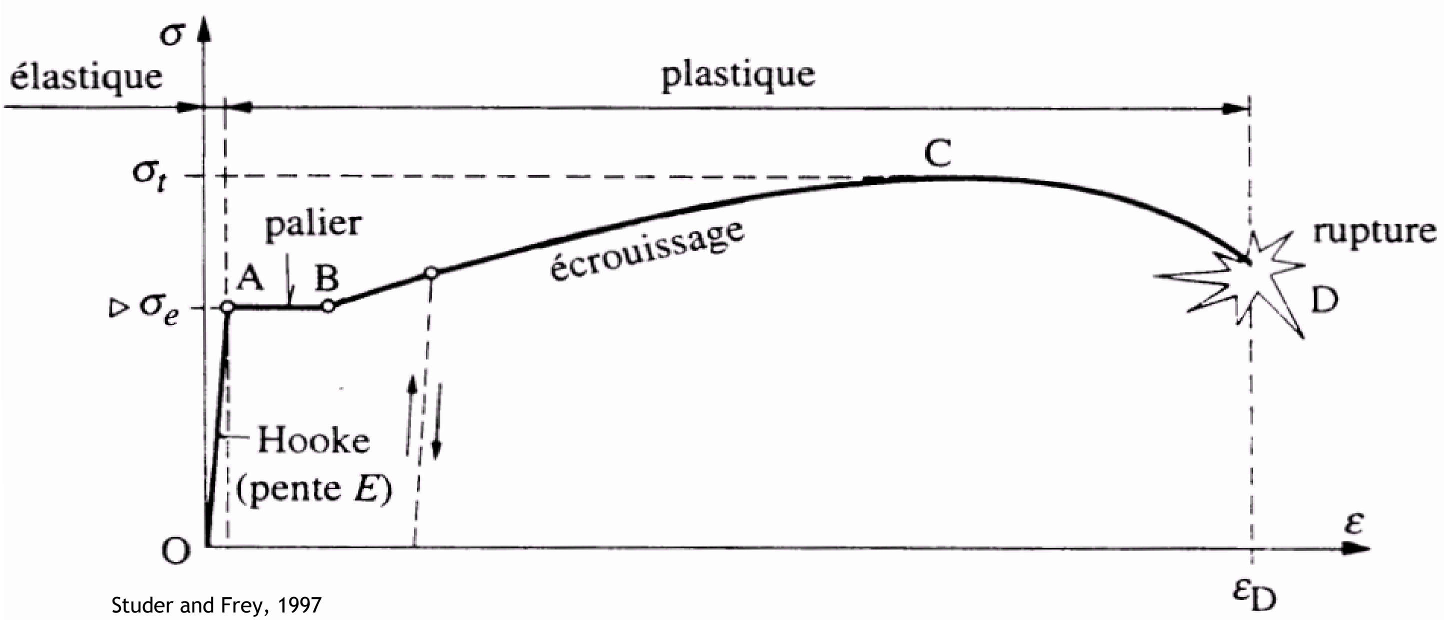


Plasticity characteristics

Hardening

Glide of cristallographic planes impeded by dislocations

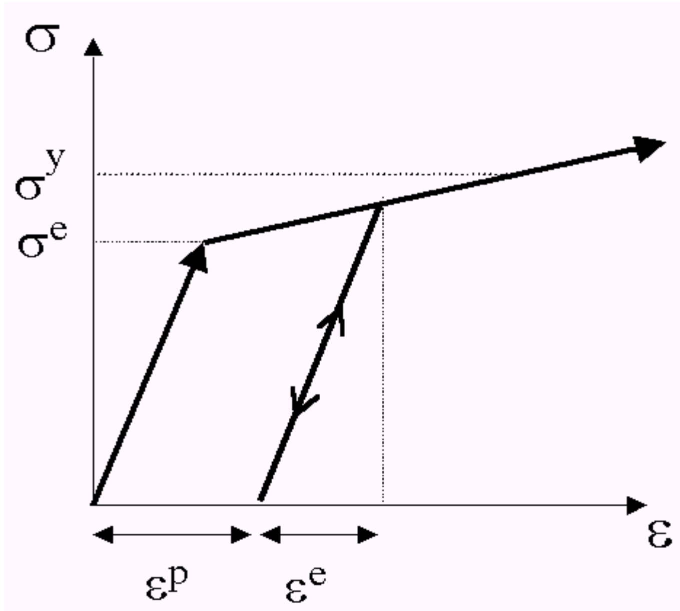
Increase of σ needed to produce further plastic (permanent) strains



Studer and Frey, 1997

Between B and C, there is an extension of the set of admissible stress states that can occur without further permanent strain increase

Uniaxial plasticity



Strain partition $\varepsilon = \varepsilon^e + \varepsilon^p$

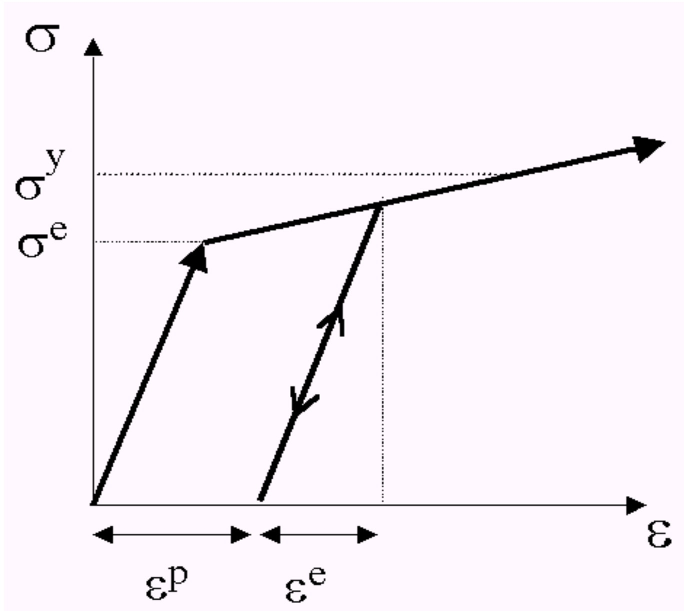
The plasticity criterion has to be expressed in stress space due to the partition of strains

$f^p = \sigma - \sigma_y(\kappa) = 0$ → Increase of ε^p

$f^p = \sigma - \sigma_y(\kappa) < 0$ → Elastic behaviour

$f^p = \sigma - \sigma_y(\kappa) > 0$ → Non admissible states

Uniaxial plasticity



Constitutive relation

$$\sigma = E\varepsilon^{e} = E\left(\varepsilon - \varepsilon^{p}\right)$$

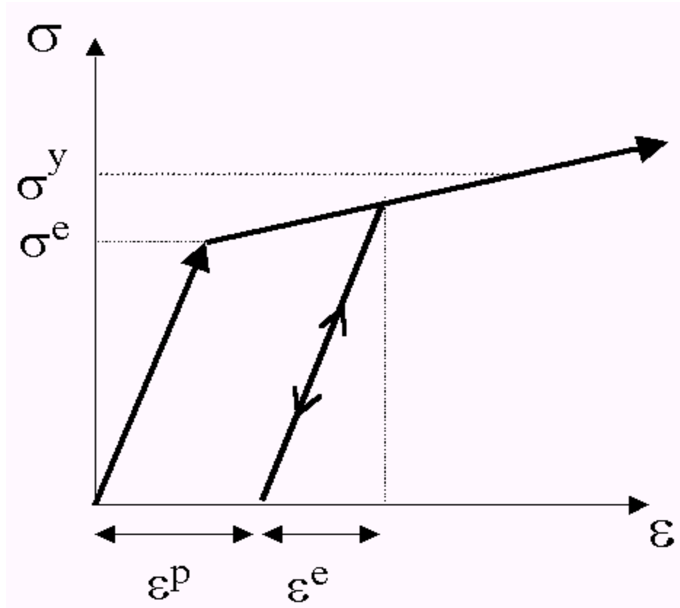
Consistency condition

The point representing the stress state in the stress space has to remain on the reversible domain when plastic strains are increasing

Hardening

If the stress increases, the boundary of the admissible domain f^{p} adapts and the admissible domain expands (if $\sigma_{y}\left(\kappa\right)$ is an increasing function)

Uniaxial plasticity



Hardening parameter κ

History parameter representing the cumulated effect of plastic dissipation

Irreversibility quantification

Strain hardening $\delta\kappa \div \delta\varepsilon^p$

Work hardening $\delta\kappa \div \sigma\delta\varepsilon^p$

Hardening law $\sigma_{hardening} = f(\kappa)$

Multiaxial plasticity - yield surface

Yield surface in the space of stresses

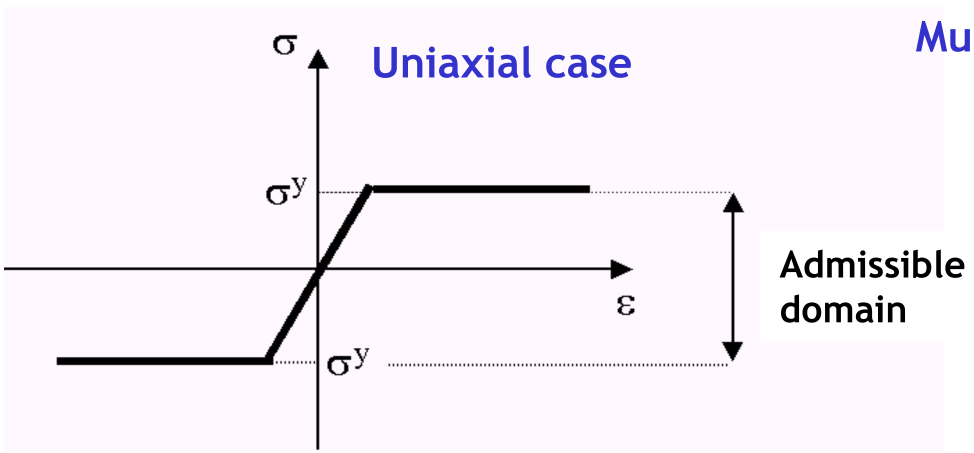
A stress state is defined by principal values $(\sigma_1, \sigma_2, \sigma_3)$

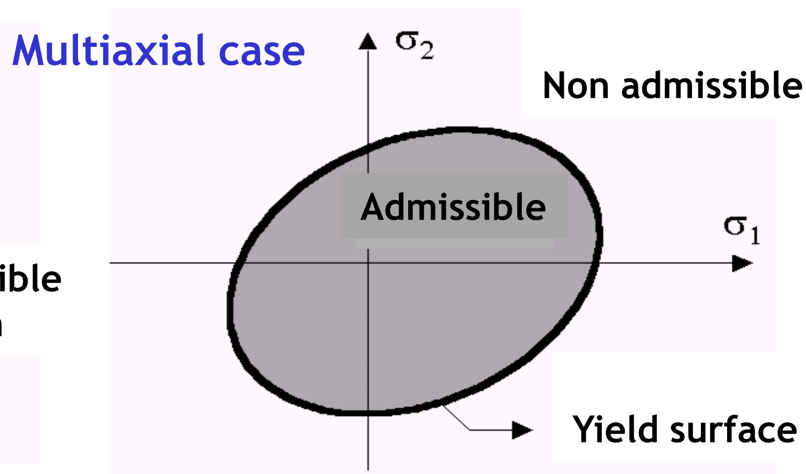
The domain of admissible stresses is limited in the state of the material

Shape of such a domain (for an isotropic material)

$$f^p = \sigma_{eq}\left(\sigma_1, \sigma_2, \sigma_3\right) - \sigma_y\left(\kappa\right) = 0$$

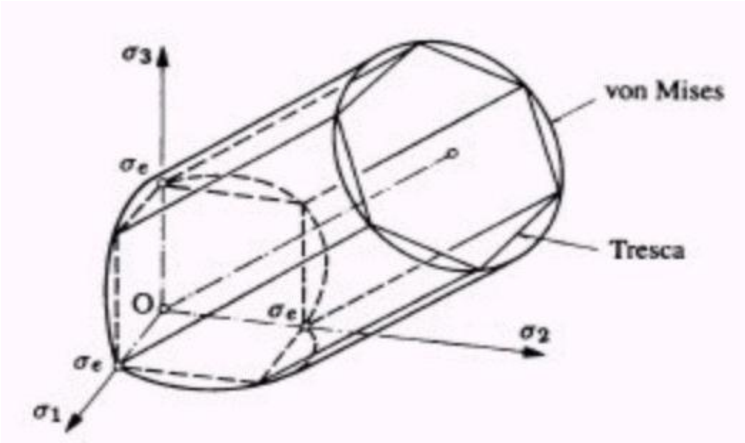
The shape of the domain quantifies the influence of the different components of σ_{ij}



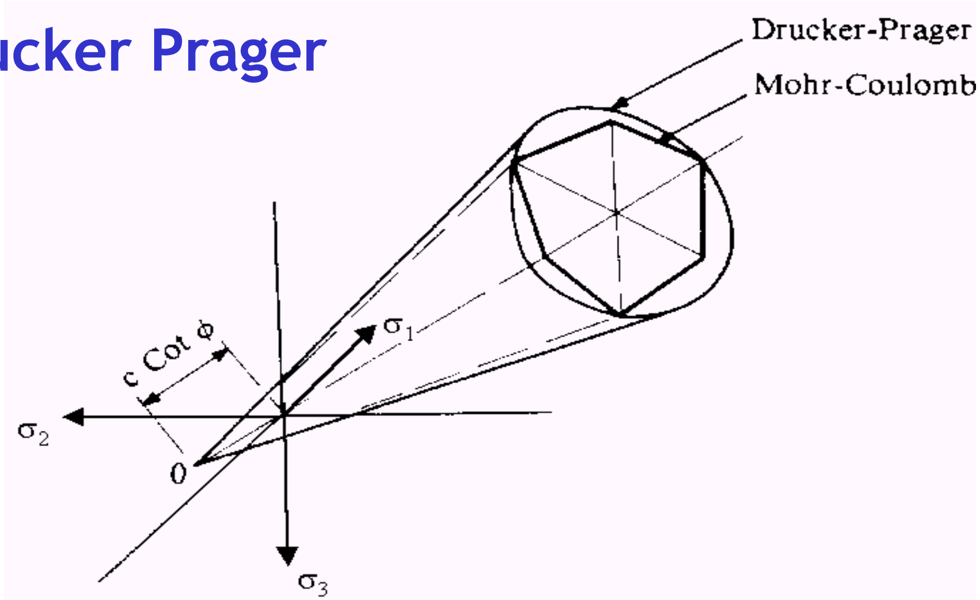


Multiaxial plasticity - criteria examples (invariants)

Tresca - von Mises



Coulomb - Drucker Prager

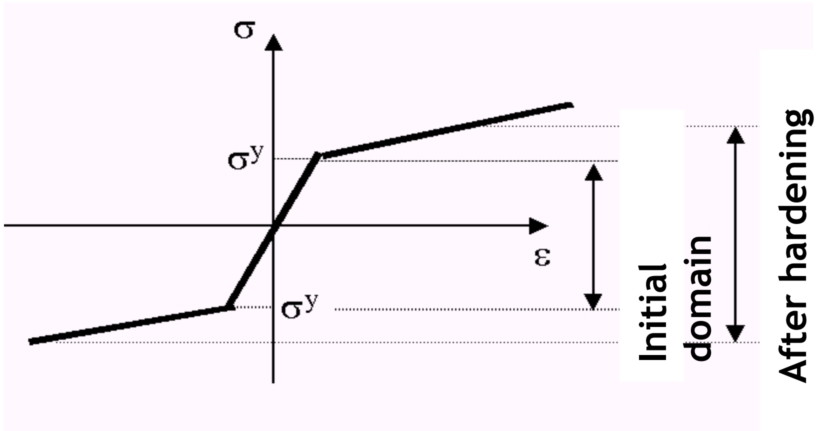


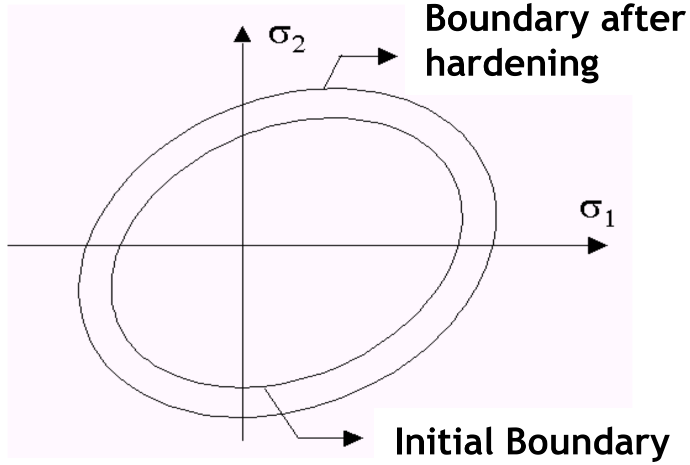
Owen and Hinton, 1980

Multiaxial plasticity - hardening

Isotropic hardening

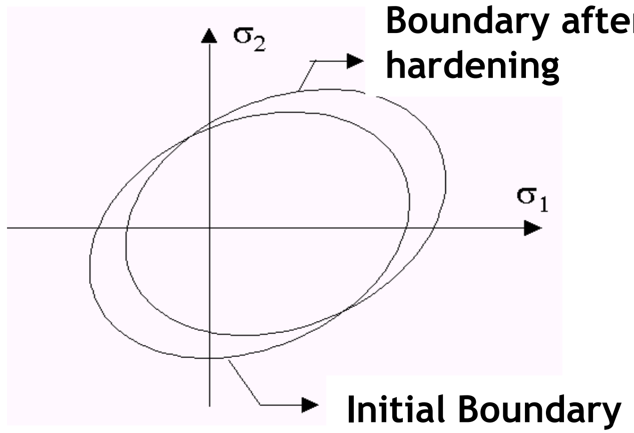
Expansion of the domain of admissible stresses





Kinematic hardening (multiaxial)

Translation of the domain of admissible stresses



Plasticity - multiaxial case

Partition of strains $\{\varepsilon\} = \{\varepsilon^e\} + \{\varepsilon^p\}$

Plasticity criterion

$f^p = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\kappa) = 0$ → Increase of ε^p

$f^p = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\kappa) < 0$ → Elastic behaviour

$f^p = \sigma_{eq}(\sigma_1, \sigma_2, \sigma_3) - \sigma_y(\kappa) > 0$ → Non admissible state

'Direction' of plastic strains $\{d\varepsilon^p\} = d\lambda \left\{ \frac{\partial g^p}{\partial \sigma} \right\}$

g^p is called the plastic potential function

$g^p = f^p$ most often chosen for metals

$g^p \neq f^p$ Required for soils, ... when dilatancy occurs

Plasticity - multiaxial case

Consistency condition

$$df^p = \left\{ \frac{\partial f^p}{\partial \sigma} \right\} \{d\sigma\} + \frac{\partial f^p}{\partial \kappa} d\kappa = 0$$

Constitutive relation

$$\{\sigma\} = [H]\{\varepsilon^e\} = [H](\{\varepsilon\} - \{\varepsilon^p\})$$

Hardening law

$$\sigma_{hardening} = f(\kappa)$$

Choice of strain hardening - work hardening law

Boils down to choice of $d\kappa = f\,(d\lambda)$

Plasticity - applications

[http://club.quizkerala.com/wp-content/uploads/2010/04/524.jpg]

[unknown source]

Progressive collapse

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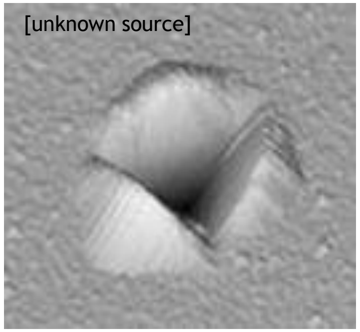
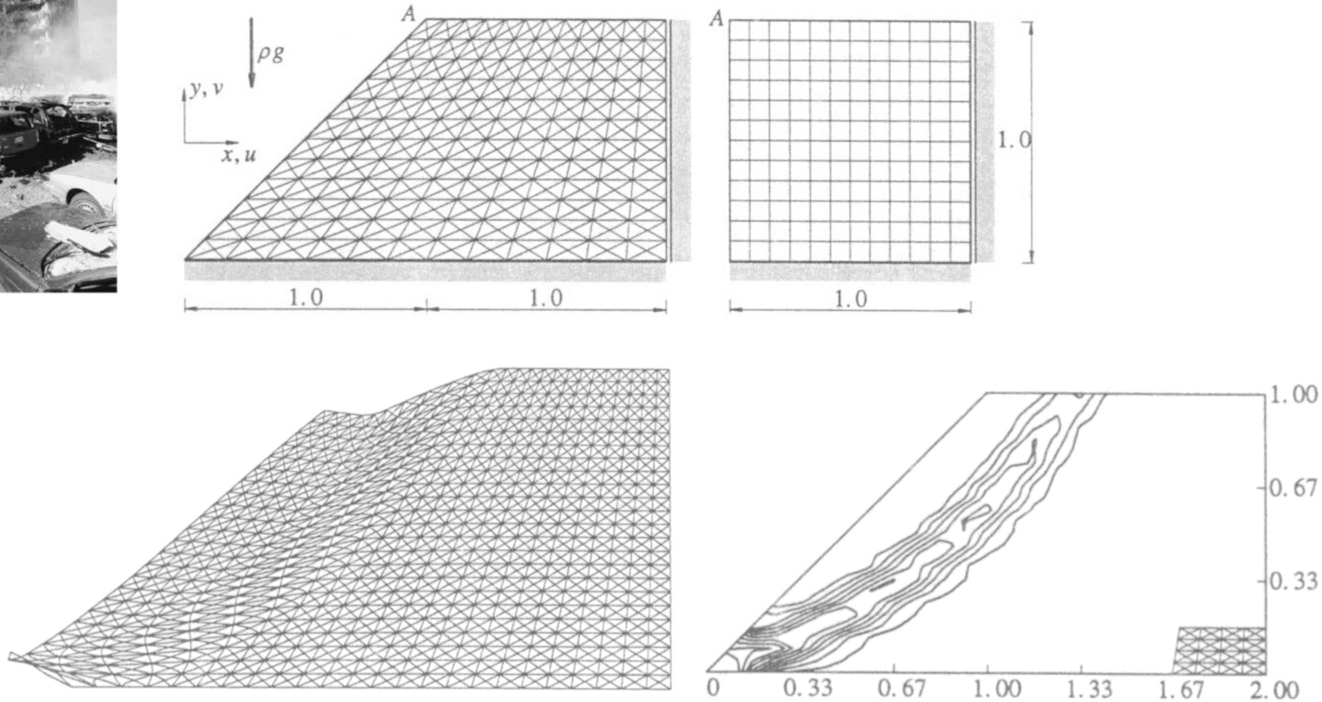


Nanoindentation

Slope stability



Ph.D. Jerzy Pamin (TUDelft, 1994)

Plastic law with 'nonlocal' effects

Drucker-Prager criterion (low resistance in tension)