Applying Nonlinear Analysis to Learn the Fundamentals of Structural Stability

Course Overview – By using nonlinear structural analysis software as the basis for a virtual laboratory, students will explore and learn the fundamentals of structural stability. Per European terminology, methods of analysis reviewed and employed in this course include linear buckling analysis (LBA) as well as geometric nonlinear analysis (GNA), material nonlinear analysis (MNA), and geometric and material nonlinear analysis (GMNA), and their counterparts that include initial imperfections (GNIA, MNIA, and GMNIA). The stability of members, such as columns and beams, and systems are explored.

Lecturer: Ronald D. Ziemian, PhD, PE
Professor, Bucknell University, Lewisburg, PA USA

Software: MASTAN2 (available at www.mastan2.com at no cost)

Lecture 1 – An Introduction to Elastic and Inelastic Analyses
After reviewing the finite element method as means for analyzing two- and three-dimensional frames and trusses, a concentrated plasticity (plastic hinge) model will be introduced as a means for accounting for material nonlinear behavior. Students will employ first-order elastic and inelastic analyses of a simple structural system to comprehend basic concepts. The impact of axial force on the plastic strength of members will be demonstrated.

Lecture 2 – Geometric Nonlinear Analysis
The basic concepts of Lecture 1 will expanded to include geometric nonlinear behavior. Using a similar hands-on approach, second-order elastic behavior will be explored, which will then be modified to include material nonlinear behavior. Next, an explanation and investigation of elastic and inelastic critical load (bifurcation by eigenvalue) analyses will be completed. The lecture will conclude by studying a two-dimensional frame to illustrate the first- and second-order elastic and inelastic analysis capabilities reviewed.

Lectures 3 and 4 – Behavior of Compression Members
This lecture will focus on fully understanding the behavior compression members, such as columns in building or chord and web members in a truss bridge. Using the analysis capabilities learned in Lectures 1 and 2, a hands-on approach will be used to systematically retract the assumptions related to Euler buckling. The impact of factors such a material yielding, residual stresses, initial out-of-straightness, and support conditions will be explored.

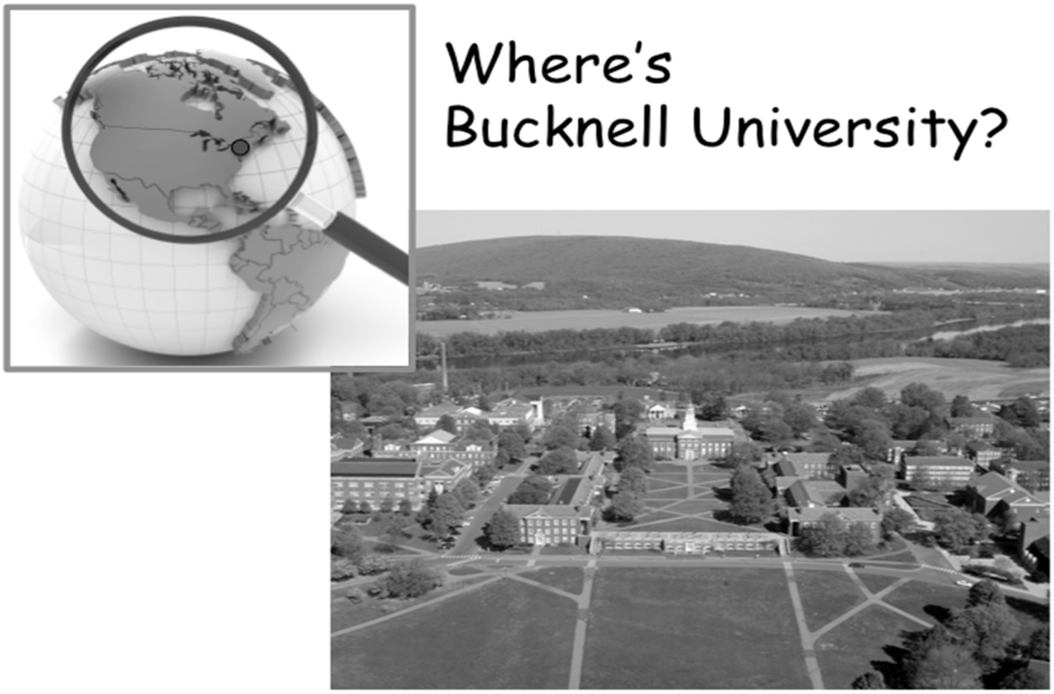
Lectures 5 and 6 – Behavior of Flexural Members
This lecture will focus on understanding the behavior flexural members, such as beams in a building or girders in bridge. Continuing with a hands-on approach, the

strength limit states of beams, including full yielding and in/elastic lateral torsional buckling, will be explored. The impact of factors such a material yielding, residual stresses, initial out-of-straightness, lateral bracing, and moment gradient will be studied.

Lecture 7 – Behavior of Beam-Columns and Structural Systems
With the basics now in hand, this lecture will explore the behavior of members and systems with members subject to the combined effects of compression and flexure. Students will compare hand methods for approximating geometric nonlinear effects with results obtained using rigorous second-order computational analysis. The lecture will conclude with an overview on how some international specifications permit the use of advanced methods of nonlinear analysis (GMNIA) to design steel structures.

About the Speaker
Ron Ziemian is a Professor of Civil and Environmental Engineering at Bucknell University in Lewisburg, PA, USA. He received his BSCE, MENG, and PhD degrees from Cornell University. In addition to authoring papers and completing research in the design and analysis of steel and aluminum structures, Dr. Ziemian is co-author of the textbook *Matrix Structural Analysis* (Wiley, 2000) and the editor for the 6th edition of the *Guide to Stability Design Criteria for Metal Structures* (Wiley, 2010). He is currently chair of the American Institute of Steel Construction's Task Committee 10 on Frame Stability, and he recently completed his terms as chair of the Structural Stability Research Council and chair of AISC's Task Group on Inelastic Analysis and Design. He serves on the AISC and Aluminum Association Specification Committees and is active with the Steel Joist Institute. Dr. Ziemian, with W. McGuire and G. Deierlein, were awarded the ASCE Norman Medal (1994) for their paper on employing advanced methods of inelastic analysis in the limit states design of steel structures, and he was the recipient of the AISC Special Achievement Award (2006) for his innovative development of the advanced structural analysis MASTAN2 software and his key role in its use to develop the fully-revised 2005 AISC Specification provisions for stability analysis and design of steel structures. In April 2013, Dr. Ziemian received the ASCE Shortridge Hardesty Award for his "substantial accomplishments in research, service, and teaching, as well as advancing practice in the field of structural stability." He has also received Bucknell University's *Presidential Award for Teaching Excellence (2000)*, and in 2010 was named a Bucknell University Presidential Professor.

Where's Bucknell University?



Applying Nonlinear Analysis to Learn the Fundamentals of Structural Stability

Ron Ziemian

13-Aug-2014 to 14-Aug-2014

PROPEC

UFOP

Course Overview

- Employ a virtual laboratory to learn basic concepts of structural stability
- Seven 90-minute lectures
  - Lectures 1 & 2 Introduction to Nonlinear Analysis
  - Lectures 3 & 4 Behavior of Compression Members
  - Lectures 5 & 6 Behavior of Flexural Members
  - Lecture 7 Beam-columns and Structural Systems
- Software employed is MASTAN2 which is available at no cost at www.mastan2.com

Better designs will come from a better understanding of behavior

G. Winter, W. McGuire, T. Pekoz, A. Nilson, J. Abel, P. Gergely, R. White, T. Ingrafea

$$U = \sum_{i=1}^{\infty} Experiences_i$$

Introduction to Nonlinear Analysis

Ron Ziemian

Lectures 1 & 2: 13-Aug-2014

PROPEC UFOP

The function of a structural engineer is to design — not to analyze

Norris and Wilbur
1960

Analysis is a means to an end rather than the end itself.

Role of the analysis:

- forces, moments and deflections --> design equations
- insight into the behavior of a structure
  --> better the understanding, better the design

Limit States Design:

Prior to limit of resistance, significant nonlinear response, including

- geometrical effects (P-Δ, P-δ)
- material effects (yielding, cracking, crushing)
- combined effects

Impetus:

AISC Ch. C: P-Δ, P-δ (App. 7)
App. 1: Inelastic Analysis
AS4100 Section 4, App. D,E,F
App. D: Advanced Analysis
Seismic: Pushover Analysis
Other: Progressive Collapse

Limit States Design

Nonlinear Analysis

Available Software

Education

Research

Computers and Structures Inc. <education@csiamerica.com>
To: Ron Ziemian
Reply-To: education@csiamerica.com
Performance Based Design Seminar in Los Angeles

May 7, 2014 2:30 AM

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A One-Day Technology Transfer Event
The theory and practice of
PERFORMANCE-BASED DESIGN
THE FUTURE OF EARTHQUAKE ENGINEERING

A SEMINAR FOR EVERY PRACTICING ENGINEER
BRIDGING THE GAP BETWEEN RESEARCH AND PRACTICE

Performance-based design is a major shift from traditional structural design concepts and represents the future of earthquake engineering. The procedure provides a method for determining acceptable levels of earthquake damage. Also, it is based on the recognition that yielding does not constitute failure and that preplanned yielding of certain members of a structure during an earthquake can actually help to save the rest of the structure. In this technology-packed seminar, Ashraf will present the theory and practical application of nonlinear analysis and performance-based design in terms and analogies that are very familiar to the practicing structural engineer. Attendees will leave the seminar empowered with a clear understanding of this new technology.

Nonlinear Analysis

- Hand methods
  - Second-order effects (<u>focus of later lectures</u>)
    - i.e. Moment Amplification Factors (B1 and B2 factors)
  - Material nonlinear effects
    - i.e. plastic analysis (upper and lower bound theories)
- Computer Methods (<u>focus of these lectures</u>)
  - Lots of variations
    - all use same basic concepts (most important to today)
    - one approach will be presented (basis for MASTAN2)
- Please keep in mind
  - All methods are approximate
  - Not a substitute, but a complement to good engineering

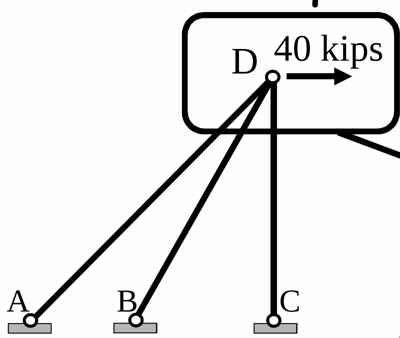
Lecture Overviews

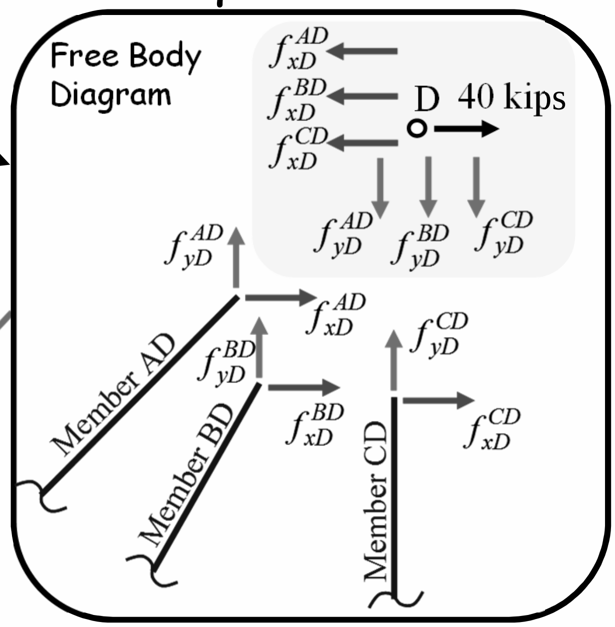
- Lecture 1
  - Brief Introduction (done!)
  - Computer Structural Analysis (Review?)
  - Basis for Material Nonlinear Models
- Lecture 2
  - Incorporating Geometric Nonlinear Behavior
  - Critical Load Analysis
  - Summary and Concluding Remarks

How does the computer get these results?

- State-of-the-Art Crystal Ball? Not quite.
- By applying 2 requirements and 1 translator
  - Two Requirements:
    - Equilibrium (equations in terms of F's and M's, 1 per d.o.f.)
    - Compatibility (equations in terms of Δ's and θ's, 1 per d.o.f.)
  - Translator "apples to oranges"
    - Constitutive Relationship (i.e. Hooke's Law, $\sigma = E\,\epsilon$)
    - Generalized to Force-to-Displacement (i.e. $F=k\Delta$)
    - Re-write equilibrium eqs. in terms of unknown displacements
- # of Equil. Eqs. = # of Unknown Displacements

Equilibrium Equations





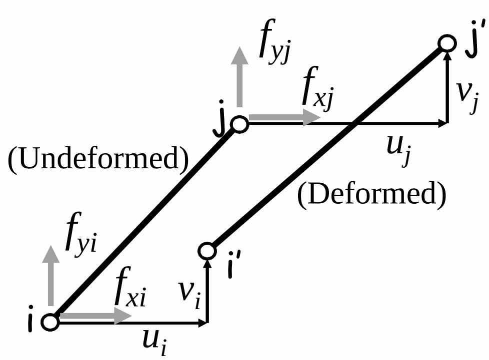
x-d.o.f. u_D : $\sum F_x = 0$

$$40 = f_{xD}^{AD} + f_{xD}^{BD} + f_{xD}^{CD}$$

y-d.o.f. v_D : $\sum F_y = 0$

$$0 = f_{yD}^{AD} + f_{yD}^{BD} + f_{yD}^{CD}$$

Translator: Forces → Displacements

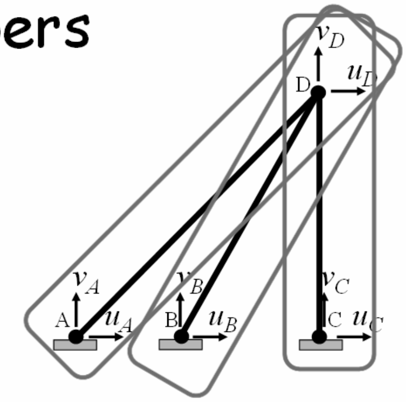


$$f_{xi} = k_{11}u_i + k_{12}v_i + k_{13}u_j + k_{14}v_j$$
$$f_{yi} = k_{21}u_i + k_{22}v_i + k_{23}u_j + k_{24}v_j$$
$$f_{xj} = k_{31}u_i + k_{32}v_i + k_{33}u_j + k_{34}v_j$$
$$f_{yj} = k_{41}u_i + k_{42}v_i + k_{43}u_j + k_{44}v_j$$

Big Question:
Where do these known stiffness coefficients k's come from?

Little Answer:
Function of member's material and geometric properties, including its orientation.

F → Δ for all members



Member AD:

$$f_{xA}^{AD} = k_{11}^{AD}u_A^{AD} + k_{12}^{AD}v_A^{AD} + k_{13}^{AD}u_D^{AD} + k_{14}^{AD}v_D^{AD}$$
$$f_{yA}^{AD} = k_{21}^{AD}u_A^{AD} + k_{22}^{AD}v_A^{AD} + k_{23}^{AD}u_D^{AD} + k_{24}^{AD}v_D^{AD}$$
$$f_{xD}^{AD} = k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}$$
$$f_{yD}^{AD} = k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}$$

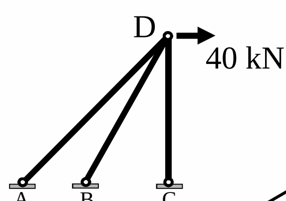
Member BD:

$$f_{xB}^{BD} = k_{11}^{BD}u_B^{BD} + k_{12}^{BD}v_B^{BD} + k_{13}^{BD}u_D^{BD} + k_{14}^{BD}v_D^{BD}$$
$$f_{yB}^{BD} = k_{21}^{BD}u_B^{BD} + k_{22}^{BD}v_B^{BD} + k_{23}^{BD}u_D^{BD} + k_{24}^{BD}v_D^{BD}$$
$$f_{xD}^{BD} = k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}$$
$$f_{yD}^{BD} = k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}$$

Member CD:

$$f_{xC}^{CD} = k_{11}^{CD}u_C^{CD} + k_{12}^{CD}v_C^{CD} + k_{13}^{CD}u_D^{CD} + k_{14}^{CD}v_D^{CD}$$
$$f_{yC}^{CD} = k_{21}^{CD}u_C^{CD} + k_{22}^{CD}v_C^{CD} + k_{23}^{CD}u_D^{CD} + k_{24}^{CD}v_D^{CD}$$
$$f_{xD}^{CD} = k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}$$
$$f_{yD}^{CD} = k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}$$

Substituting into Equil. Eqs.



x-d.o.f. u_D : $\sum F_x = 0$

$$40 = f_{xD}^{AD} + f_{xD}^{BD} + f_{xD}^{CD}$$

Member AD:

$$f_{xD}^{AD} = k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}$$

Member BD:

$$f_{xD}^{BD} = k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}$$

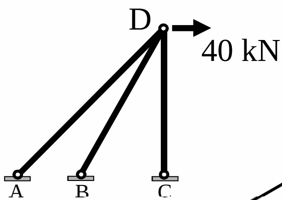
Member CD:

$$f_{xD}^{CD} = k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}$$

u_D : $\sum F_x = 0$

$$40 = \left(k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}\right) + \left(k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}\right) + \left(k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}\right)$$

Substituting into Equil. Eqs. (cont.)



Member AD:

$$f_{yD}^{AD} = k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}$$

Member BD:

$$f_{yD}^{BD} = k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}$$

Member CD:

$$f_{yD}^{CD} = k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}$$

y-d.o.f. v_D : $\sum F_y = 0$

$$0 = f_{yD}^{AD} + f_{yD}^{BD} + f_{yD}^{CD}$$

v_D : $\sum F_y = 0$

$$\begin{aligned} 0 = &\left(k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}\right) + \\ &\left(k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}\right) + \\ &\left(k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}\right) \end{aligned}$$

So, where are we at?

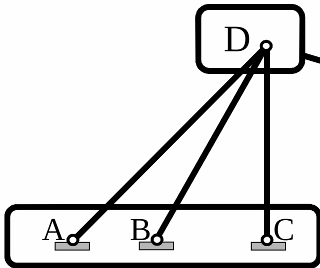
- We have two equilibrium equations (1 per d.o.f.) in terms of a <u>lot</u> of displacements:

$$\begin{aligned} u_D:\quad 40 = &\left(k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}\right) + \\ &\left(k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}\right) + \\ &\left(k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}\right) \end{aligned}$$

$$\begin{aligned} v_D:\quad 0 = &\left(k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}\right) + \\ &\left(k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}\right) + \\ &\left(k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}\right) \end{aligned}$$

What card haven't we played yet?

Compatibility Eqs. (consistent deflections)



Member-to-Support

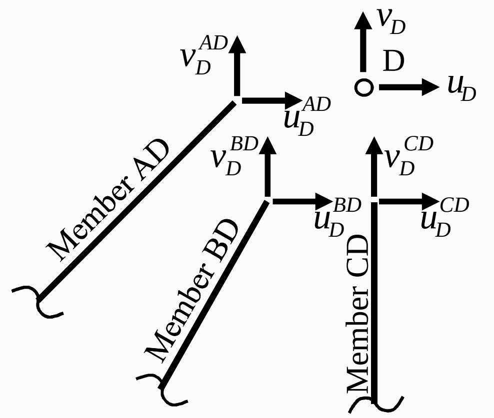
$$u_A = u_A^{AD} = 0 \qquad u_C = u_C^{CD} = 0$$

$$v_A = v_A^{AD} = 0 \qquad v_C = v_C^{CD} = 0$$

$$u_B = u_B^{BD} = 0$$

$$v_B = v_B^{BD} = 0$$

Member-to-Member



$$u_D = u_D^{AD} = u_D^{BD} = u_D^{CD}$$

$$v_D = v_D^{AD} = v_D^{BD} = v_D^{CD}$$

Time for some serious simplifying

- Applying Compatibility to Equil. Eqs.:

$$
\begin{aligned}
u_D:\quad 40 = {} & \left(k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}\right) + \\
& \left(k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}\right) + \\
& \left(k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}\right) \\
v_D:\quad 0 = {} & \left(k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}\right) + \\
& \left(k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}\right) + \\
& \left(k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}\right)
\end{aligned}
$$

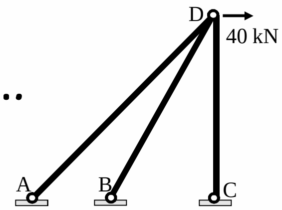
All = u_D

All = 0

All = v_D

Which simplifies to…

After simplifying…



$$u_D: \quad 40 = \left(k_{33}^{AD} + k_{33}^{BD} + k_{33}^{CD}\right)\overset{?}{u_D} + \left(k_{34}^{AD} + k_{34}^{BD} + k_{34}^{CD}\right)\overset{?}{v_D}$$

$$v_D: \quad 0 = \left(k_{43}^{AD} + k_{43}^{BD} + k_{43}^{CD}\right)\overset{?}{u_D} + \left(k_{44}^{AD} + k_{44}^{BD} + k_{44}^{CD}\right)\overset{?}{v_D}$$

Since k's are known, we have
2 Equations and 2 Unknowns

Solve for Unknown Displacements

$u_D = \#$ and $v_D = \#\#$

With all displacements, solve for member forces…

$$u_A = u_A^{AD} = 0$$
$$v_A = v_A^{AD} = 0$$

$$u_B = u_B^{BD} = 0$$
$$v_B = v_B^{BD} = 0$$

$$u_C = u_C^{CD} = 0$$
$$v_C = v_C^{CD} = 0$$

$$u_D = u_D^{AD} = u_D^{BD} = u_D^{CD} = \#$$
$$v_D = v_D^{AD} = v_D^{BD} = v_D^{CD} = \#\#$$

Member AD:

$$f_{xA}^{AD} = k_{11}^{AD}u_A^{AD} + k_{12}^{AD}v_A^{AD} + k_{13}^{AD}u_D^{AD} + k_{14}^{AD}v_D^{AD}$$
$$f_{yA}^{AD} = k_{21}^{AD}u_A^{AD} + k_{22}^{AD}v_A^{AD} + k_{23}^{AD}u_D^{AD} + k_{24}^{AD}v_D^{AD}$$
$$f_{xD}^{AD} = k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}$$
$$f_{yD}^{AD} = k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}$$

Member BD:

$$f_{xB}^{BD} = k_{11}^{BD}u_B^{BD} + k_{12}^{BD}v_B^{BD} + k_{13}^{BD}u_D^{BD} + k_{14}^{BD}v_D^{BD}$$
$$f_{yB}^{BD} = k_{21}^{BD}u_B^{BD} + k_{22}^{BD}v_B^{BD} + k_{23}^{BD}u_D^{BD} + k_{24}^{BD}v_D^{BD}$$
$$f_{xD}^{BD} = k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}$$
$$f_{yD}^{BD} = k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}$$

Member CD:

$$f_{xC}^{CD} = k_{11}^{CD}u_C^{CD} + k_{12}^{CD}v_C^{CD} + k_{13}^{CD}u_D^{CD} + k_{14}^{CD}v_D^{CD}$$
$$f_{yC}^{CD} = k_{21}^{CD}u_C^{CD} + k_{22}^{CD}v_C^{CD} + k_{23}^{CD}u_D^{CD} + k_{24}^{CD}v_D^{CD}$$
$$f_{xD}^{CD} = k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}$$
$$f_{yD}^{CD} = k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}$$

Summary of Computer Approach

- For each d.o.f., write an equilibrium equation:

$$F_{external} = \Sigma\, f_{member} \quad \text{(Equil. Eqs.)}$$

- Re-write (translate) each member force in terms of its end displacements (Stiffness Eqs.)

$$f_{member} = \Sigma\, k_{\,member}\, \Delta_{\,member\ end}$$

- Substitute Stiffness Eqs. into above Equil. Eqs.
- Simplify Equil. Eqs. by applying member-to-member and member-to-support compatibility conditions
- Solve n Equil. Eqs. for the n unknown displacements
- Use Stiffness Eqs. to calculate member forces
- Apply Equil. Eqs. to solve for reactions

Lot's of Questions

- So, this is how most commercial programs such as SAP2000, RISA, STAAD, etc. get the answer?
  - Yes! Known as "Direct Stiffness Method"
- So, all such programs will give the same answer?
  - Yes, as long as it is a static 1^{st}-order elastic analysis.
- Wait a minute…Is this the basic analysis procedure for the "finite element method"?
  - Yes! Bit more tricky to get k's, σ's, and ϵ's

Two Big Questions

- Where do those stiffness coefficients come from?
  - You mean the ones that relate member end forces to member end displacements?
  - Yeah, those k's ! <More to come on this>
- What happens when we go static nonlinear or even dynamic?
  - Same basic procedure, but apply loads in increments and perform a series of analyses. Then, sum incremental results.

    < Much more to come on this! >

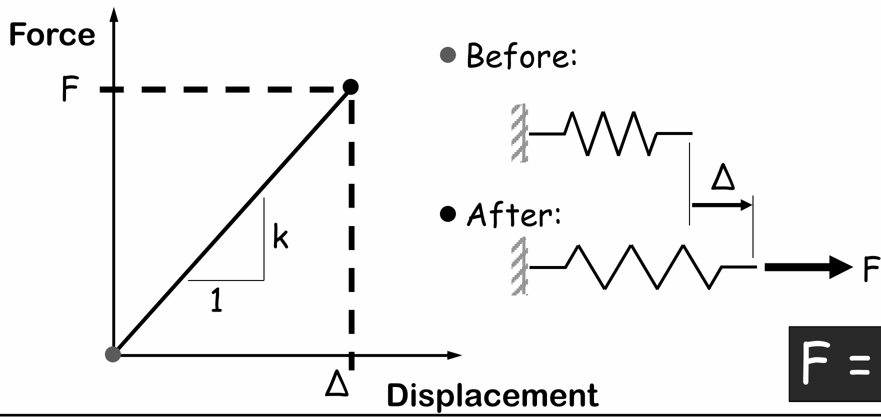
Important Points

- The only opportunity for most computer analysis software to model the actual behavior of the structure is through the member stiffness terms.
- So, to include
  - first-order effects
  - second-order effects
  - material nonlinear behavior

  Must modify member stiffness!!!
- Let's review member stiffness

Stiffness Coefficients, k's

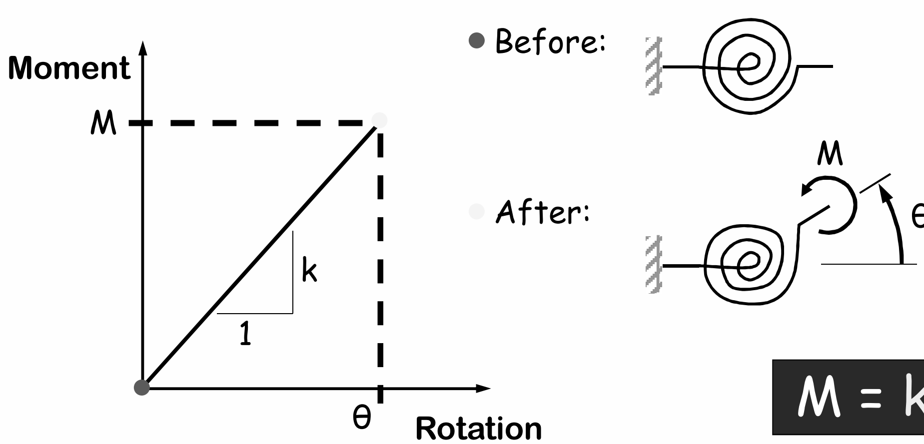
- Let's start with high school physics
  - Extension Spring Experiment



$F = k\,\Delta$

Stiffness Coefficients, k's (cont.)

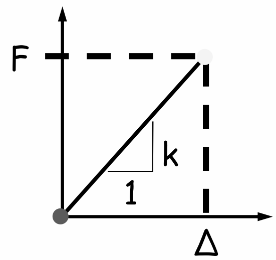
- More "advanced" high school physics lab
  - Rotational Spring Experiment



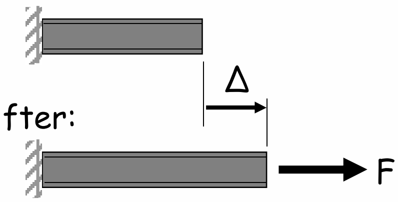
$M = k\,\theta$

How about real structural members?

- Axial force member



- Before:
- After:

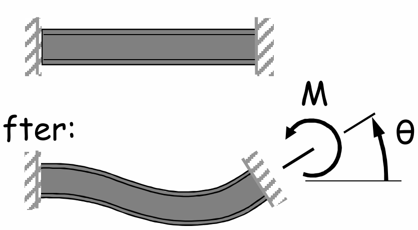


- Stiffness k function of:
  - **Geometry**: Area and Length (A↑,k↑ & L↑, k↓)
  - **Material**: Elastic Modulus (E↑,k↑)

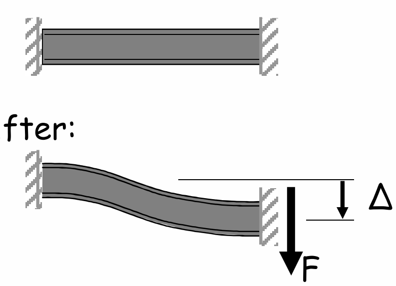
$F = k(A,L,E)\ \Delta$

How about real members? (cont.)

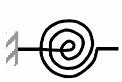
- Flexural members
  - Before:
  - After:



  - Before:
  - After:



- Stiffness k function of:
  - **Geometry**: Moment of Inertia & Length (I↑,k↑& L↑, k↓)
  - **Material**: Elastic Modulus (E↑,k↑)

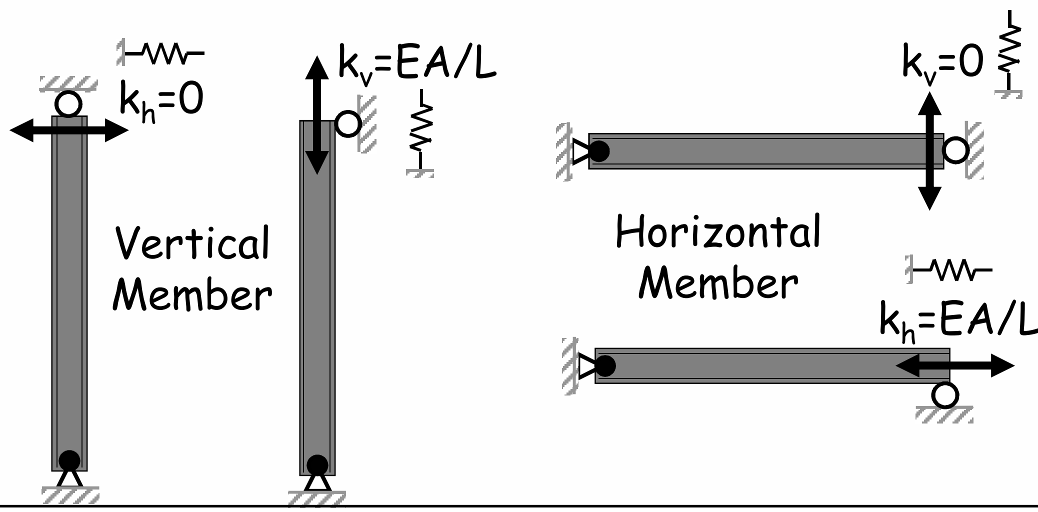


$M = k(I,L,E)\ \theta$

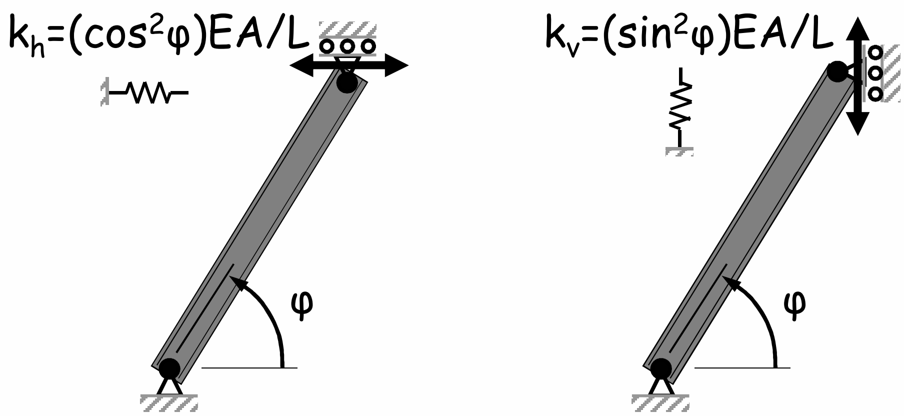
$F = k(I,L,E)\ \Delta$

Other factor impacting stiffness

- Orientation of member
  - consider axial force member:

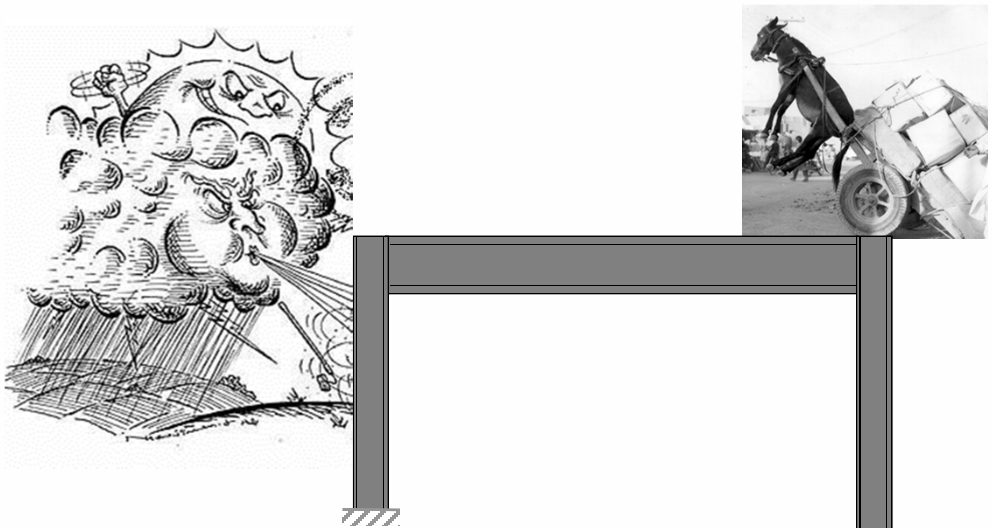


Orientation of axial force member



Important Point: Less vertical a member, the less stiffness to resist vertical loads.

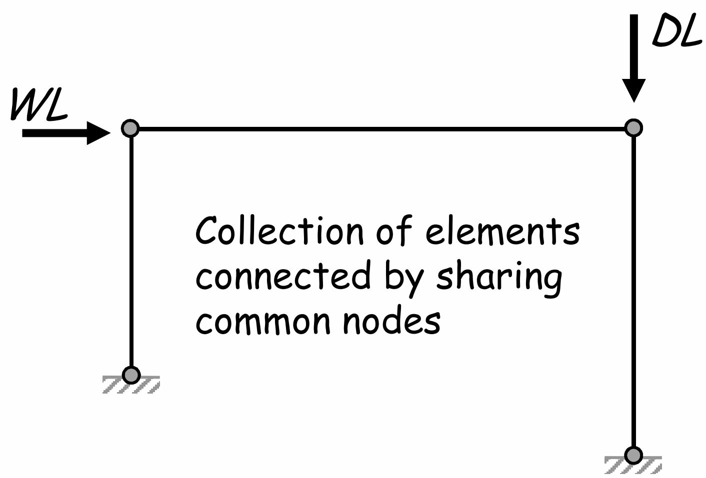
Summary: Three Perspectives



❖ Reality: What you see…

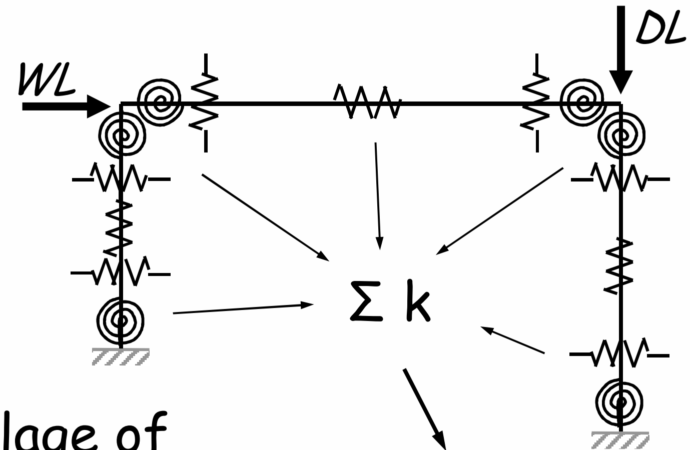
Three Perspectives (cont.)

❖ What you see on your computer screen:



Three Perspectives

- What your computer actually sees:



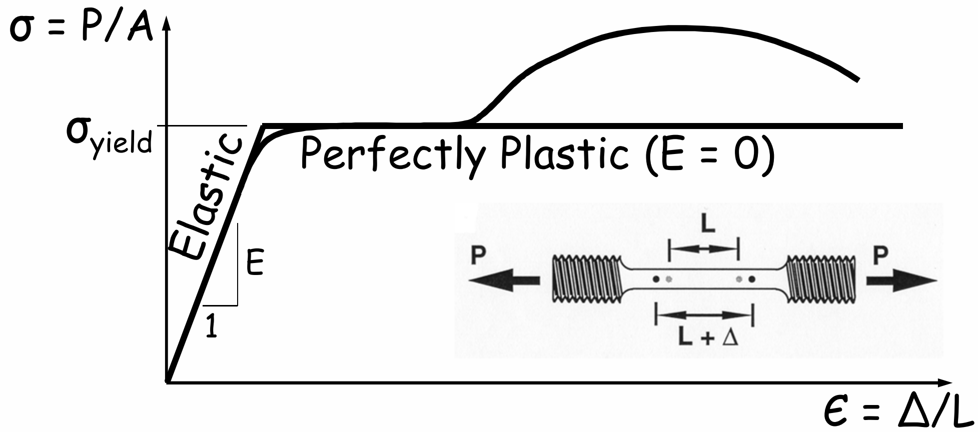
Assemblage of equivalent springs $\{F\} = [K]\{\Delta\}$

Analysis Review: Key Points

- Reviewed the “Direct Stiffness Method”
  - Equilibrium → Translator F(Δ) → Compatibility
- Response of structure controlled by stiffness of members (a.k.a. springs)
- First-order elastic stiffness of member function of:
  - Material Property (E)
  - Geometric Properties (A, I, L, and orientation)
- Time to go nonlinear…
  let’s begin with material nonlinear

Material Nonlinear (Inelastic)

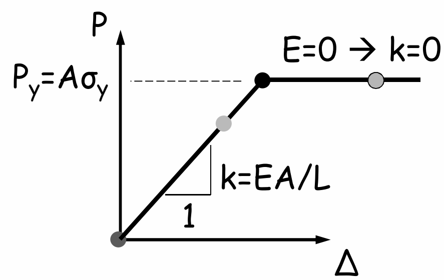
- Best place to start is with a tensile test

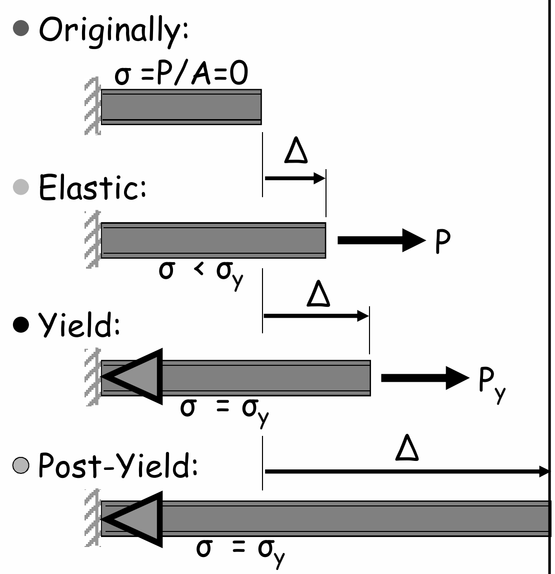


Normal Stress: Structural Members

- For typical structural steel members (L/d>10), elastic/inelastic behavior controlled by normal stresses σ's acting along the length axis of the member.
- Normal stress produced by:
  - Axial force (P/A)
  - Major and/or minor axis flexure (Mc/I)
  - Combination of above effects (i.e. P/A + Mc/I)
  - Warping (not today!)
- We will assume elastic-perfectly-plastic material (often done for steel)

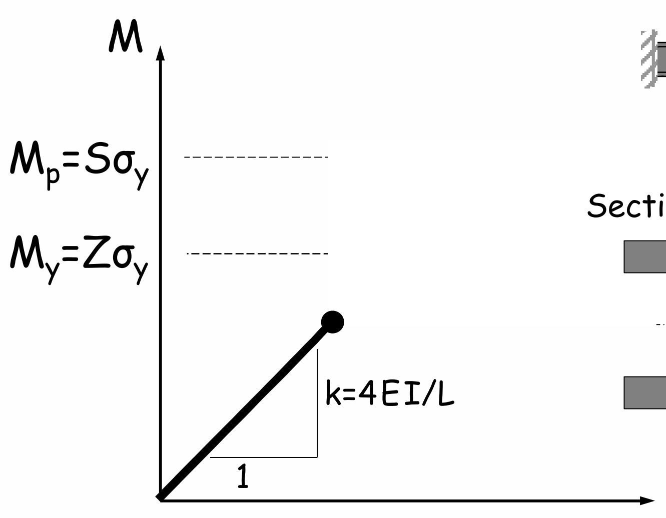
Inelastic Behavior: Axial Force

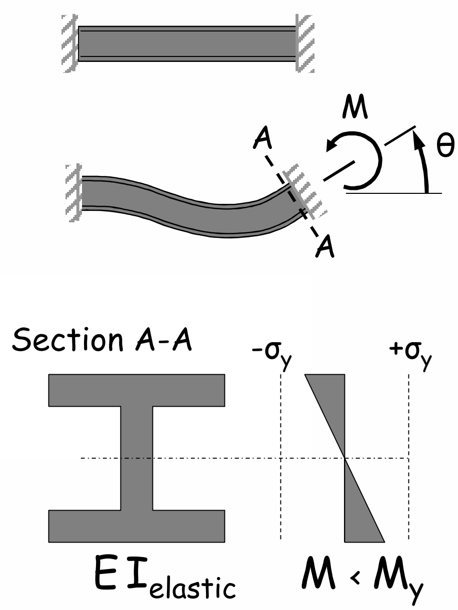




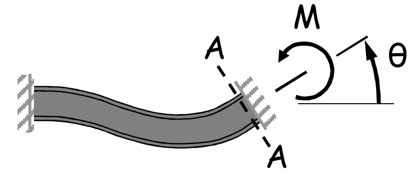
Plastic Hinge at $P = P_y$ or when $P/P_y = 1.0$

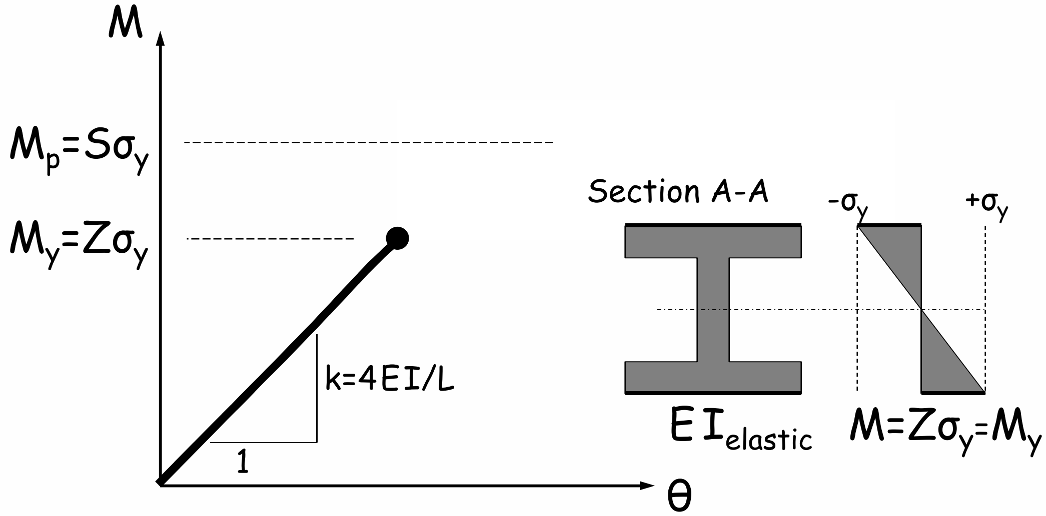
Inelastic Behavior: Flexure



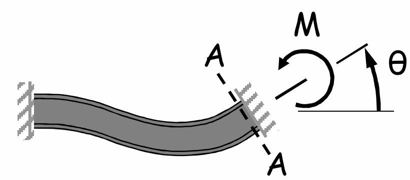


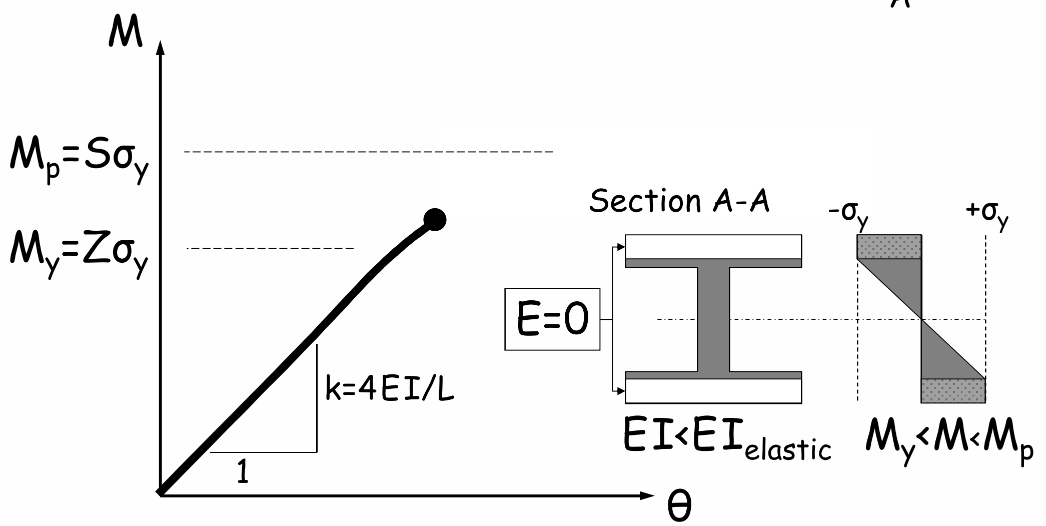
Inelastic Behavior: Flexure (cont.)



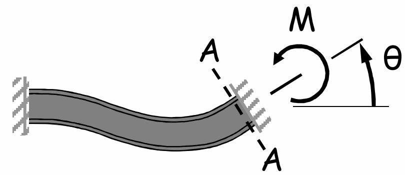


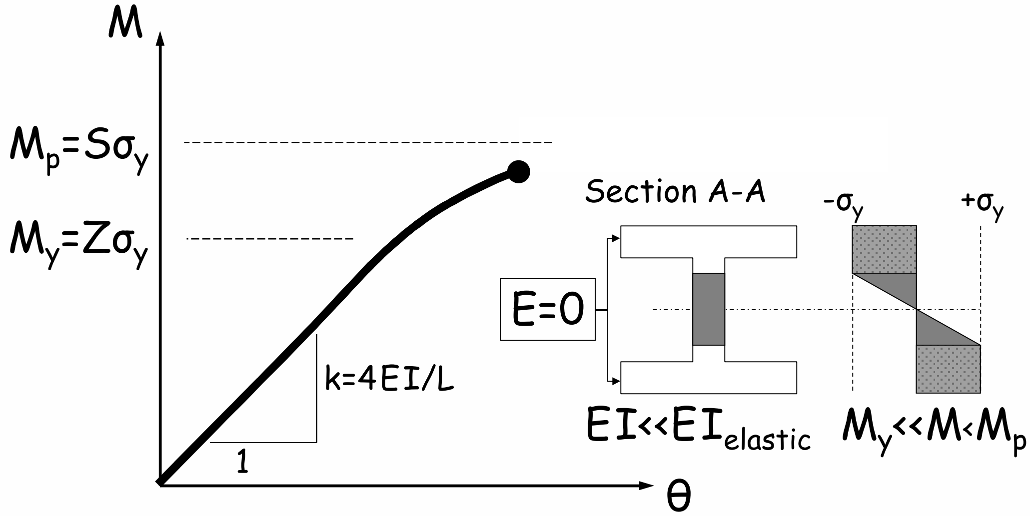
Inelastic Behavior: Flexure (cont.)



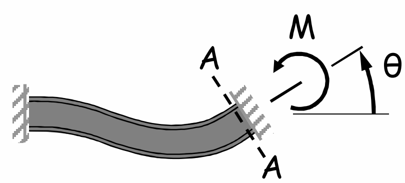


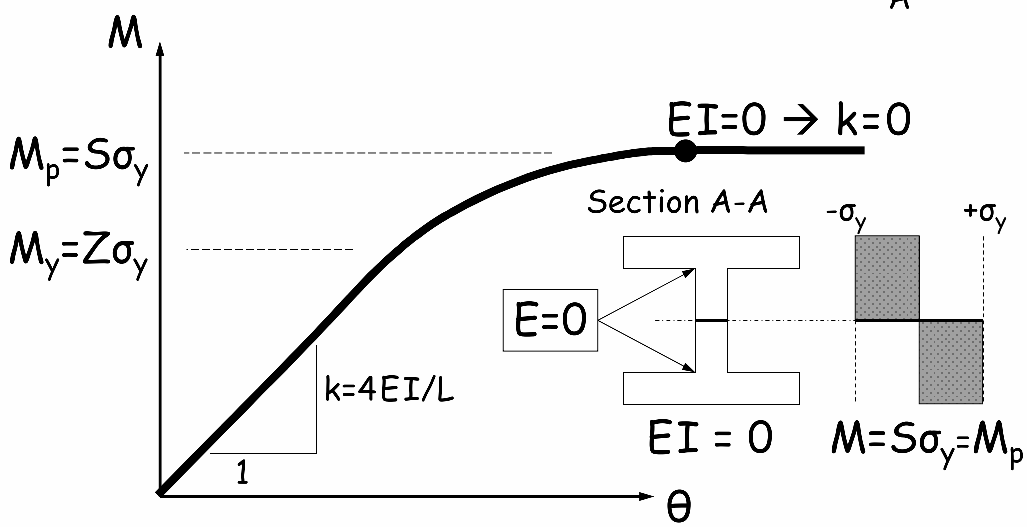
Inelastic Behavior: Flexure (cont.)



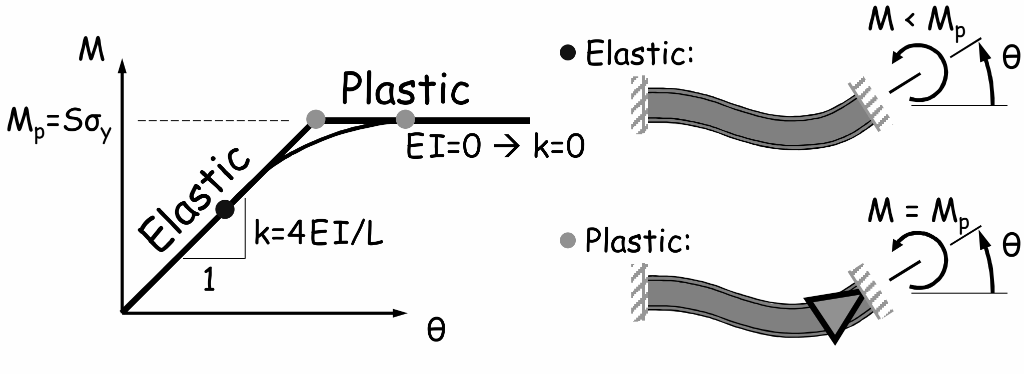


Inelastic Behavior: Flexure (cont.)





Inelastic Behavior: Flexure

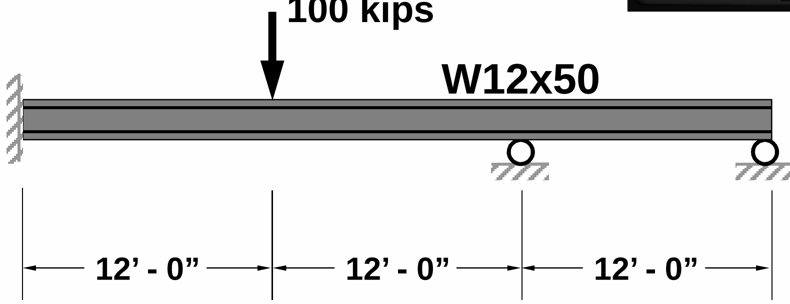


- Plastic Hinge Model – Assume section as fully elastic or fully plastic (neglect partial yielding)
- Plastic Hinge at $M = M_p$ or when $M/M_p = 1.0$

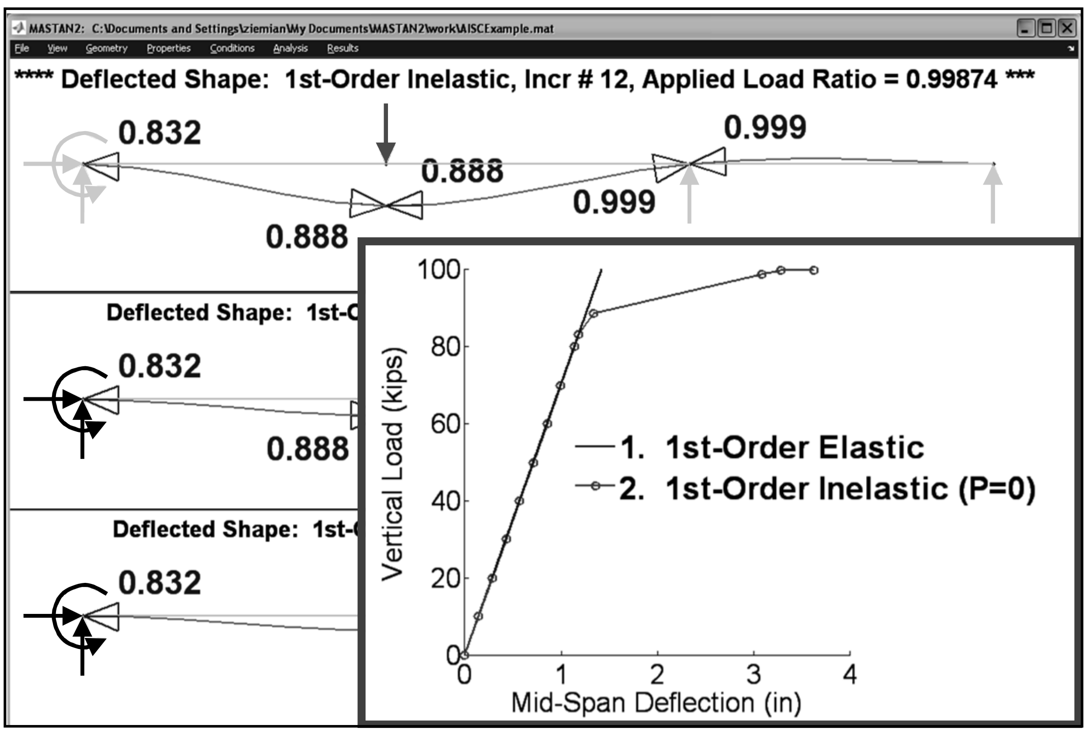
Types of inelastic models

- We will employ a plastic hinge model
  - A.K.A. "Concentrated Plasticity"
  - Section is fully elastic or fully yielded
  - Plastic hinges only at element ends
- Distributed plasticity (still line elements)
  - A.K.A. "Plastic Zone"
  - Captures gradual yielding through depth and along length
  - More accurate, but computationally more $$
- Finite element with continuum elements ($$$$)

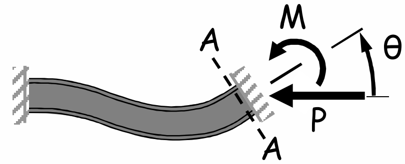
Simple Example:

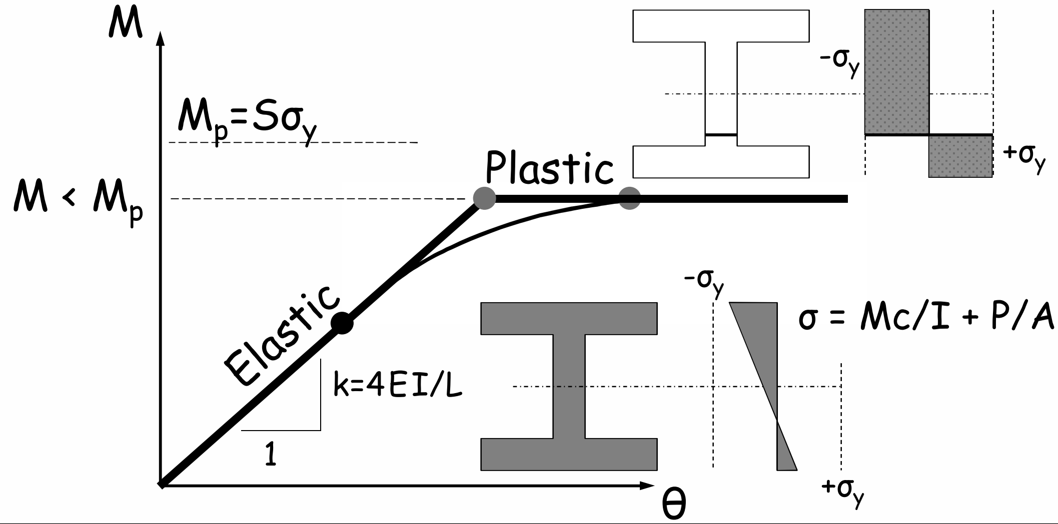


E = 29,000 ksi
σ_y = 50 ksi

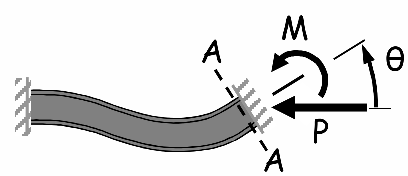


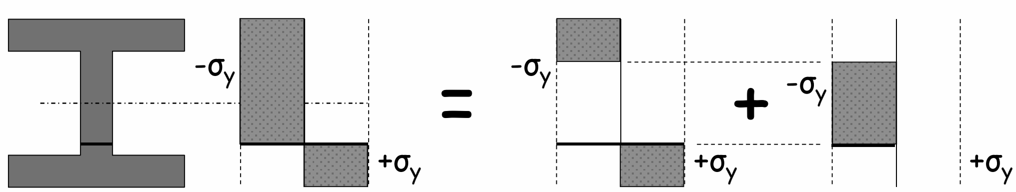
Inelastic Behavior: Combination P & M





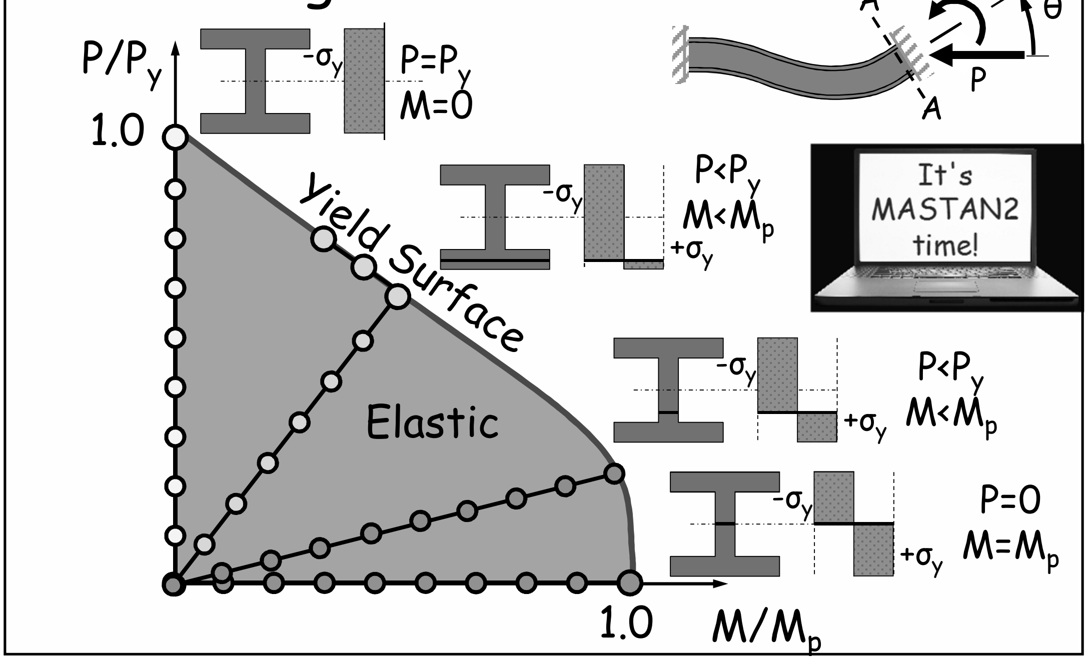
Inelastic Behavior: Combination P & M for Plastic Hinge





Fully yielded section when:	$M < M_p$ $M/M_p < 1$	$P < P_y$ $P/P_y < 1$

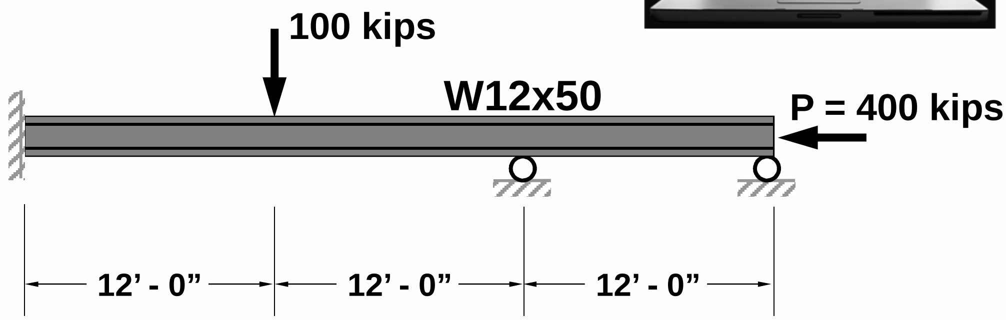
Plastic Hinge Criterion:



Material Nonlinear Analysis

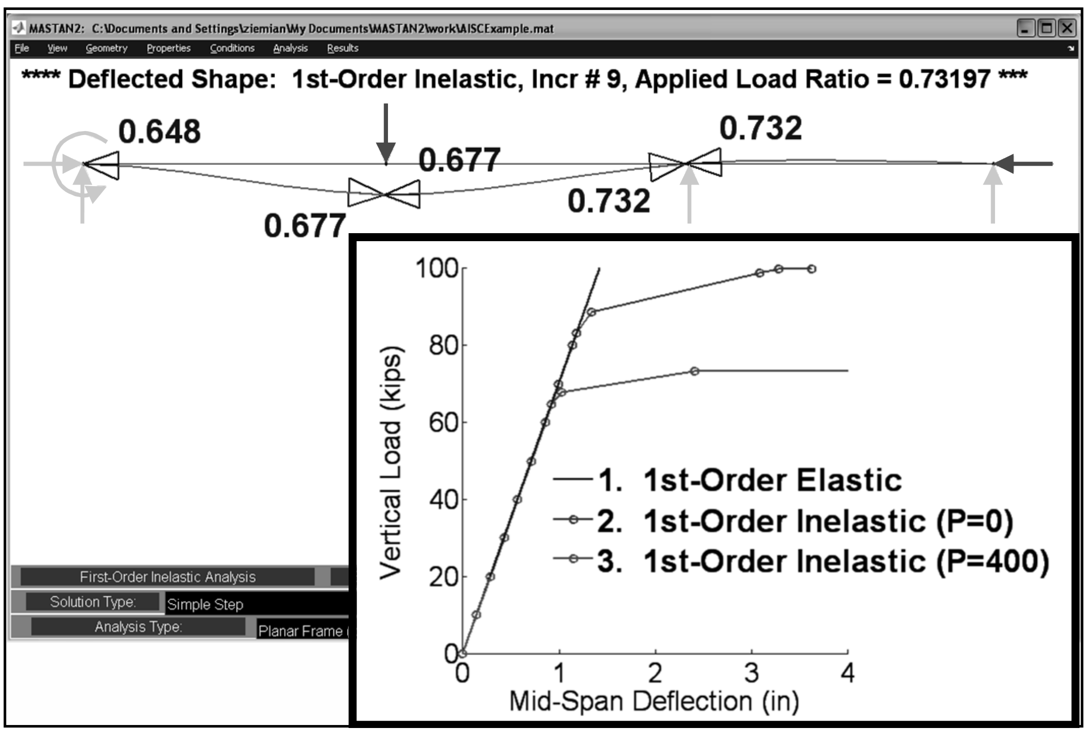
- Employ "Direct Stiffness Method" applying loads in increments: $[K]\{d\Delta\} = \{dF\}$
- During the load increment, check to see if plastic hinge(s) form. If so, scale back load increment accordingly.
- Reduce stiffness of yielded members and continue load increments
  - $k = k_{elastic} + k_{plastic}$ with $k_{plastic}$ = plastic reduction
- Continue to accumulate results of load increments until all of load is applied or a plastic mechanism forms.

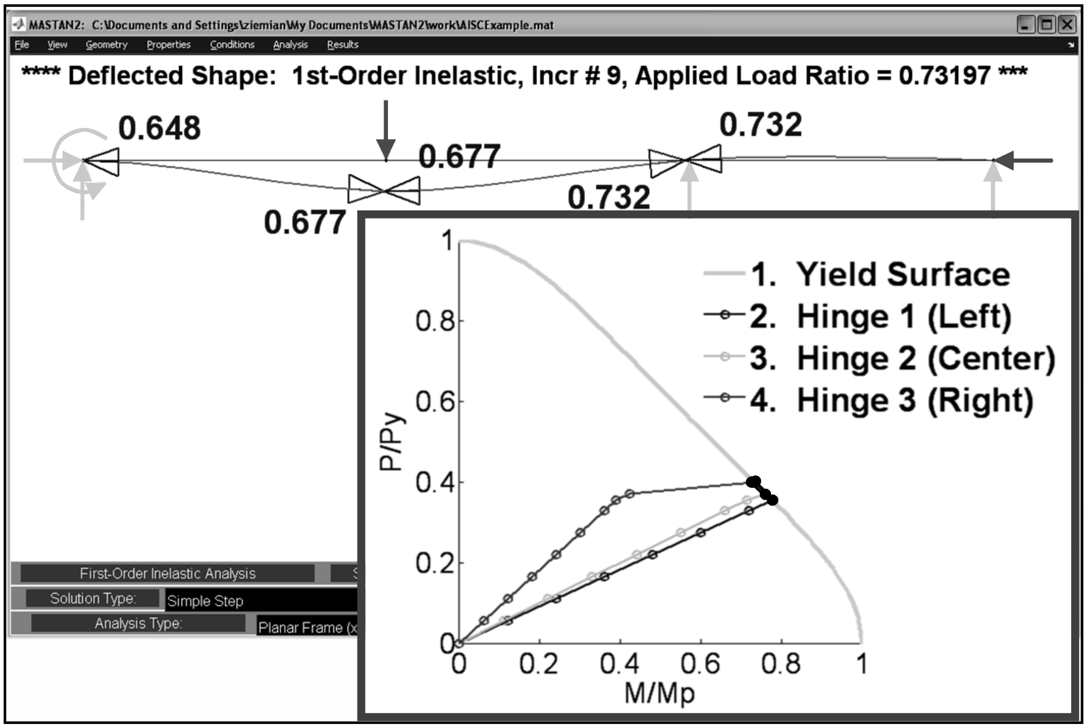
Simple Example (with axial force):



E = 29,000 ksi

σ_y = 50 ksi



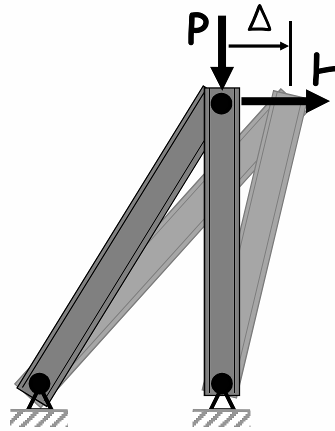


Second-Order Effects

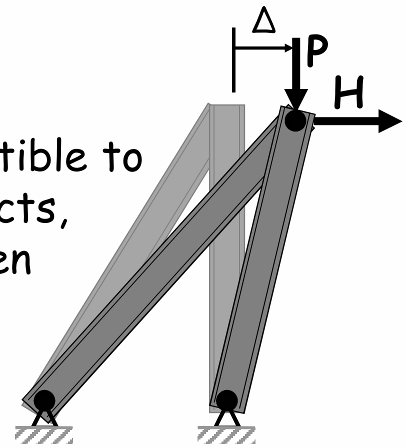
- A.K.A. "Geometric Nonlinear Behavior"
- Equilibrium Equations
  - Reality: Should be formulated on deformed shape
  - Difficulty: Deformed shape (deformations) is a function of the member forces, which are in turn a function of the deformations (Chicken 'n Egg)
  - Remedy: Perform a series of analyses with loads applied in small increments and update geometry after each load increment.

Equilibrium Equations

- Formulated on Undeformed Shape
- Formulated on Deformed Shape



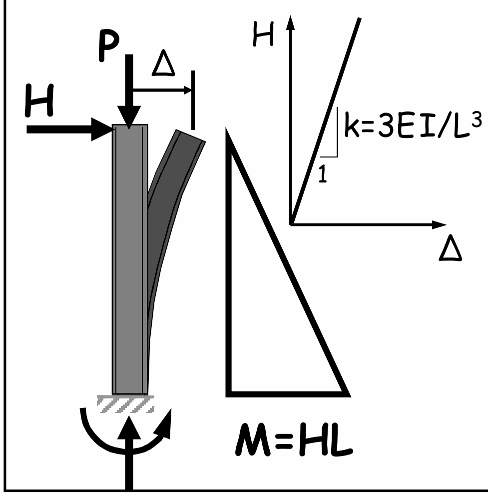
Truss is susceptible to 2nd-Order effects, luckily Δ is often quite small.

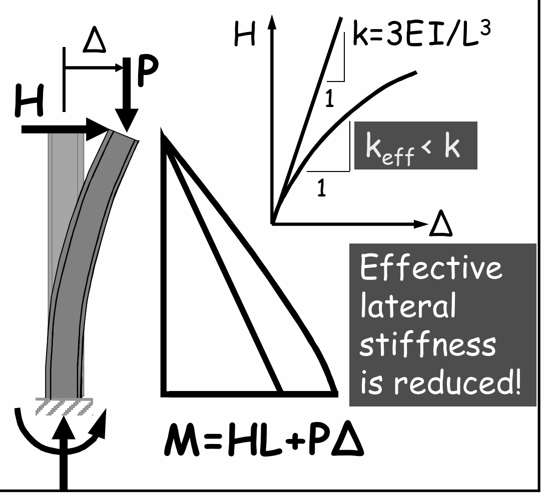


Different reactions and member forces.

Equilibrium Equations

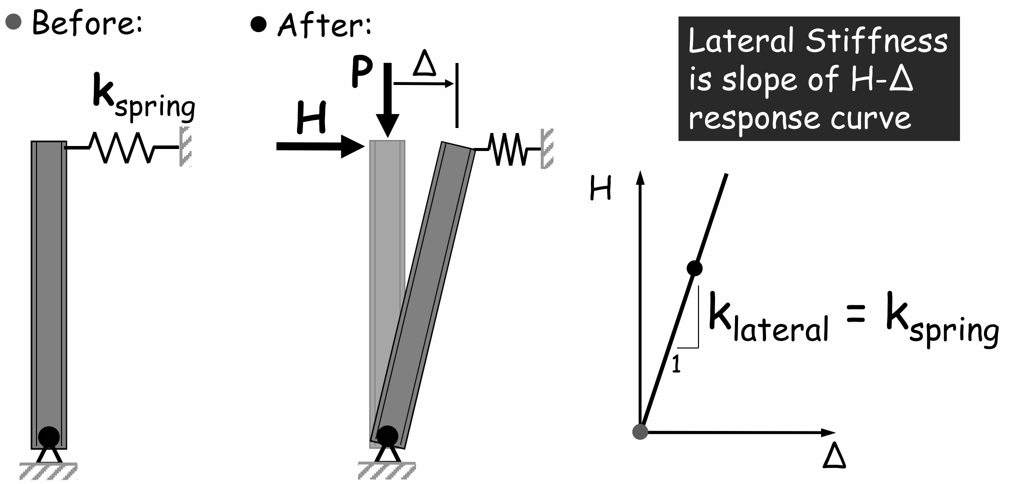
- Formulated on Undeformed Shape
- Formulated on Deformed Shape





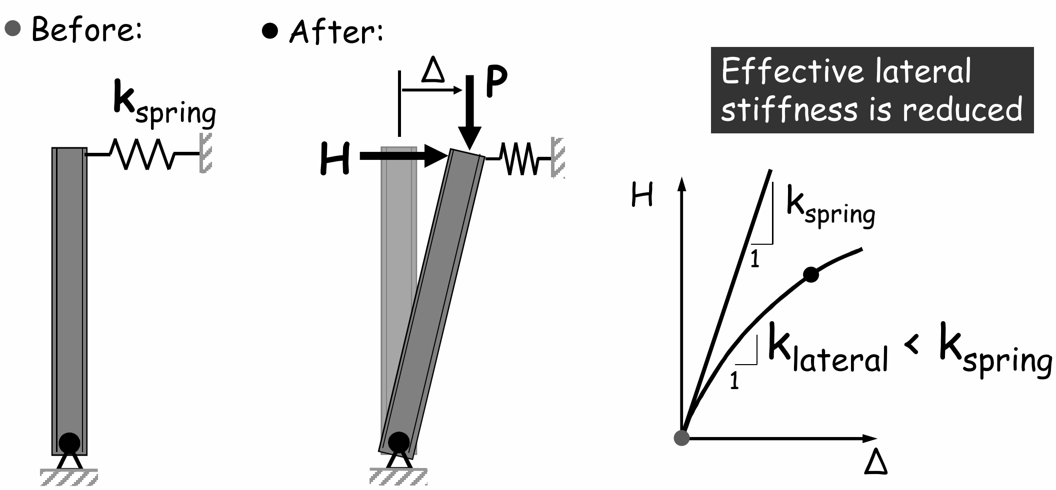
Focus on Lateral Stiffness

- Formulated on Undeformed Shape: <u>Linear Response</u>



Focus on Lateral Stiffness (cont.)

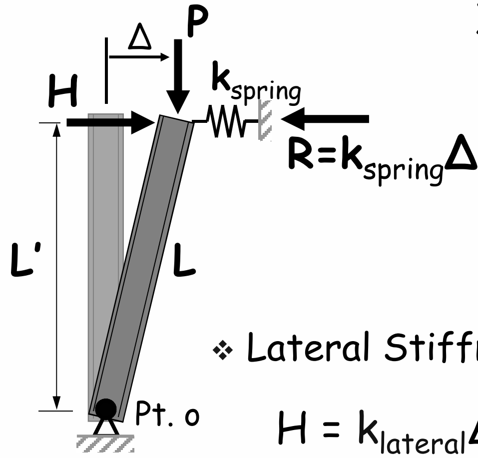
- Formulated on Deformed Shape: <u>Nonlinear Response</u>



Focus on Lateral Stiffness (cont.)

- Equilibrium Formulated on Deformed Shape

Let's start by assuming $L' \approx L$,



$\Sigma M_o = 0$ $\quad RL = HL + P\Delta$

$R = H + P\Delta/L$

$k_{spring}\Delta = H + P\Delta/L$

$H = k_{spring}\Delta - P\Delta/L$

$H = (k_{spring} - P/L)\,\Delta$

- Lateral Stiffness (slope of response curve)

$H = k_{lateral}\Delta$ with $\boxed{k_{lateral} = k_{spring} - P/L}$

Some thoughts here…

- This simple analysis becomes less "accurate" as Δ/L becomes large (i.e. $\Delta/L >> 1/5$)
  - Remedy: Perform an incremental analysis and update geometry after each load increment…hence, limit Δ/L in each step to some small amount
  - Keep in mind serviceability limits are often something like $\Delta/L < 1/400$
- Most importantly, $k_{lateral} = k_{spring} - P/L$ takes on the form:

$$k_{2nd\text{-}Order\ El.} = k_{1st\text{-}Order\ El.} + k_g$$

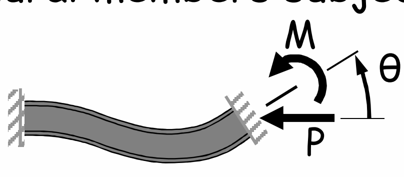
Geometric Stiffness

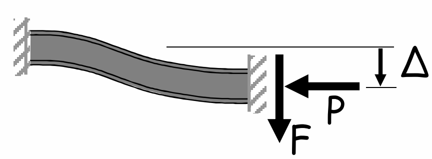
Geometric Stiffness

- Effective lateral stiffness of a member:
  - decreases as a member is compressed
    - k_g is negative for compressive P
    - backpacker example
  - increases when subjected to tension
    - k_g is positive for tensile P
    - guitar string example
- Employing geometric stiffness approach
  - Other methods exist (i.e. stability functions)

How about real members? (recall…)

- Flexural members subjected to axial force





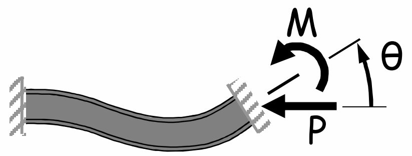
- Stiffness k function of:
  - **Geometry**: Moment of Inertia & Length (I↑,k↑& L↑, k↓)
  - **Material**: Elastic Modulus (E ↑,k↑)
  - **Axial Force**: Compressive (P↑, k↓)

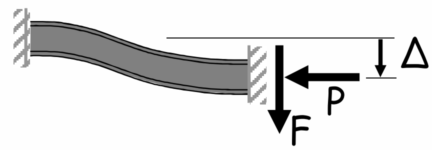
$M = k(I,L,E,P)\ \theta$

$F = k(I,L,E,P)\ \Delta$

Closer look at stiffness terms…

- Flexural members subjected to axial force





$M = k(I,L,E,P)\,\theta$ with
$k = 4EI/L - 2PL/15$

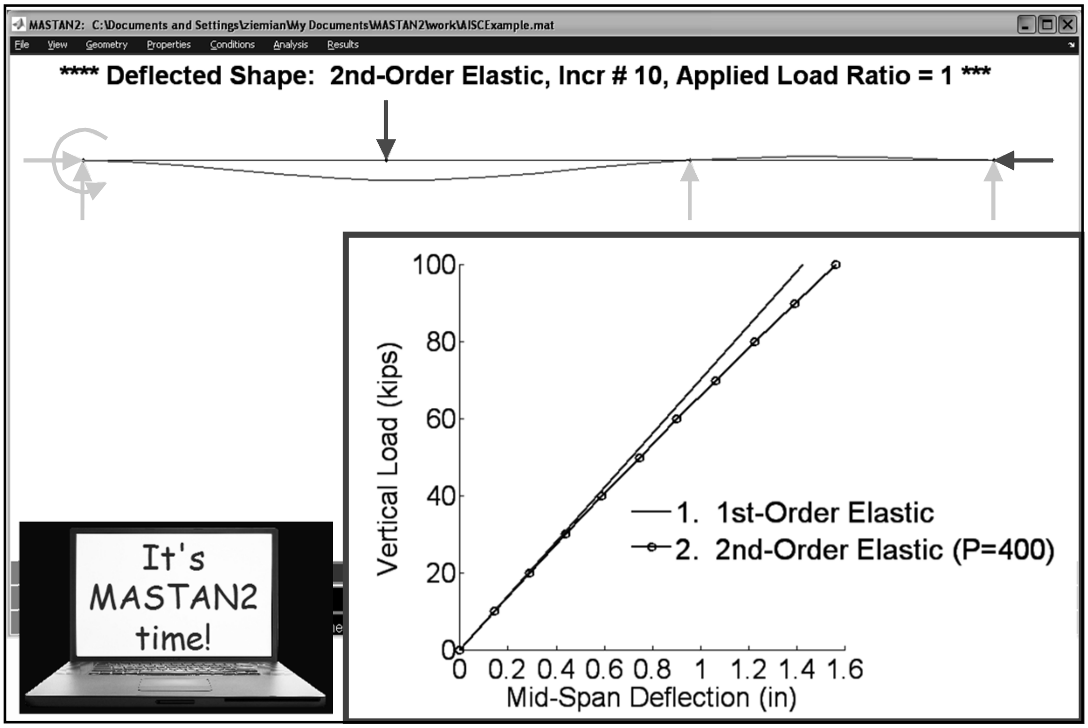
$F = k(I,L,E,P)\Delta$ with
$k = 12EI/L^3 - 6P/5L$

Again, basic form:

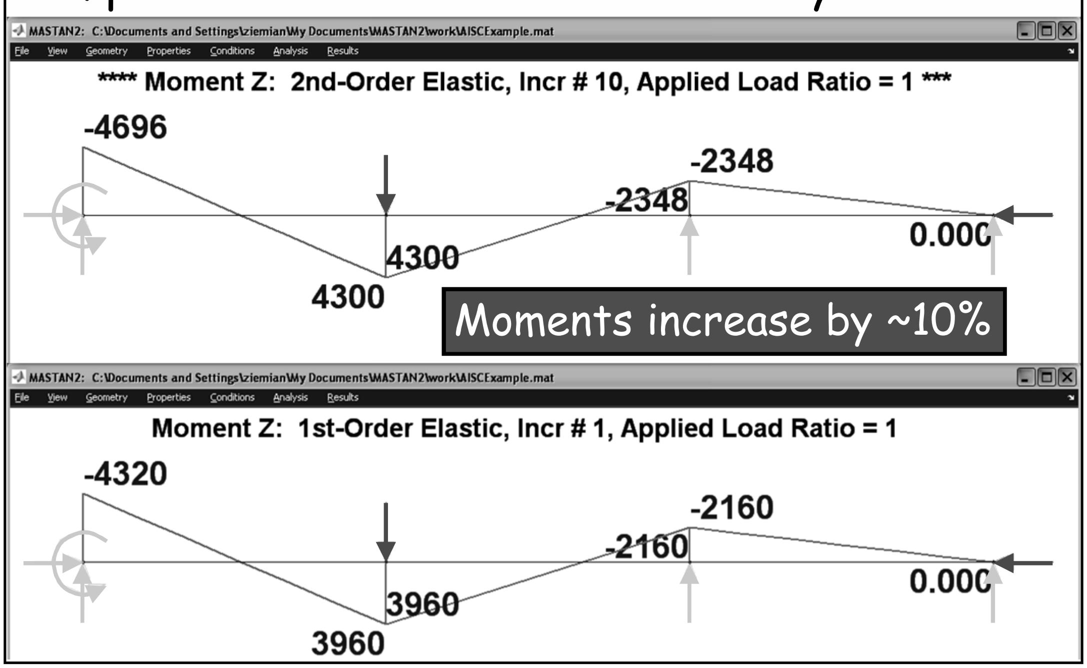
$$k_{2nd\text{-}Order\ El.} = k_{1st\text{-}Order\ El.} + k_g$$

Geometric Nonlinear Analysis

- Employ "Direct Stiffness Method" applying loads in increments: Solve Equil. Eqs. $\{dF\} = [K]\{d\Delta\}$
- At start of increment, modify member stiffness to account for presence of member forces (such as axial force):
  - $k = k_{elastic} + k_g$ with k_g = geometric stiffness
- At end of increment, update model of structural geometry to include displacements
- Continue to accumulate results of load increments ($\Delta_i = \Delta_{i-1} + d\Delta$ and $f_i = f_{i-1} + df$) until all of load is applied or elastic instability is detected.



Comparison: 1st- and 2nd-Order Analysis Results

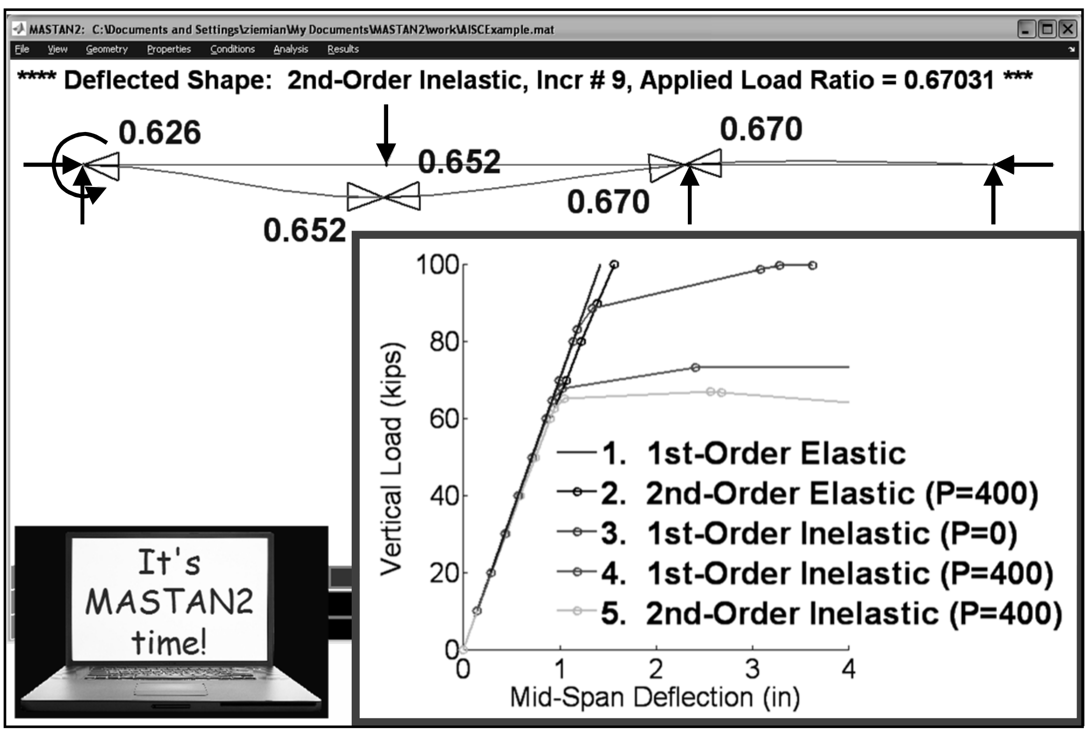


2nd-Order Inelastic Analysis

- Employ "Direct Stiffness Method" applying loads in increments: Solve Equil. Eqs. $\{dF\} = [K]\{d\Delta\}$
- At start of increment, modify member stiffness to account for presence of member forces and any yielding:

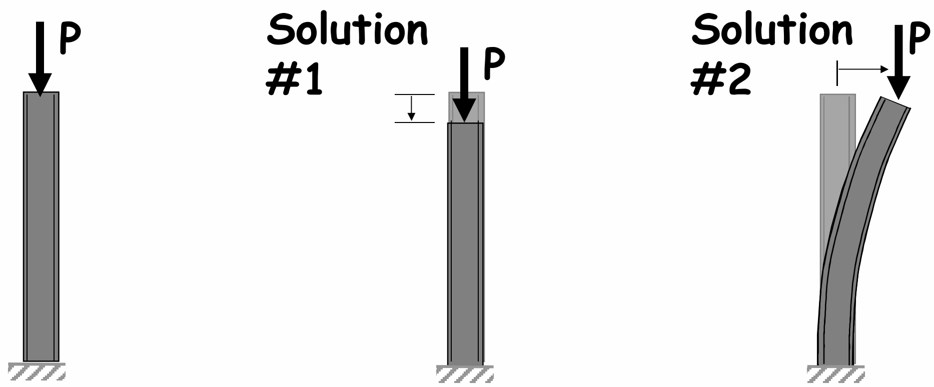
$$k = k_{elastic} + k_{geometric} + k_{plastic}$$

- At end of increment, update model of structural geometry to include displacements
- Continue to accumulate results of load increments ($\Delta_i = \Delta_{i-1} + d\Delta$ and $f_i = f_{i-1} + df$) until all of load is applied or inelastic instability is detected.



Critical Load Analysis (Basics)

- Definition: Critical or buckling load is the load at which equilibrium may be satisfied by more than one deformed shape.



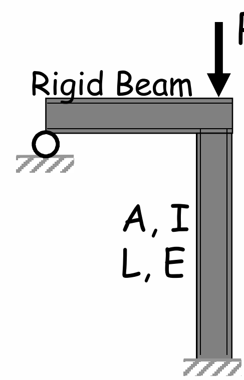
Big Q: How does computer software calculate this?

Critical Load Analysis (Background)

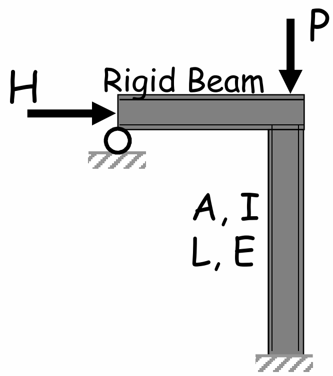
- Elastic stiffness of a member $k = k_{el} + k_g$
  - k_{el} is f(A or I, L, and E)
  - k_g is f(P,L), also note directly proportional to P
- Elastic stiffness of structure $[K] = \Sigma k$
  - $[K] = [K_{el}] + [K_g]$
  - $[K_g]$ directly proportional to applied force
    - i.e. Double applied forces, hence, double internal force distribution and double $[K_g]$
- To the computer, "buckling" will occur when our equilibrium equations $\{F\} = [K]\{\Delta\}$ permit non-unique solutions, e.g. $\det[K] = 0$.

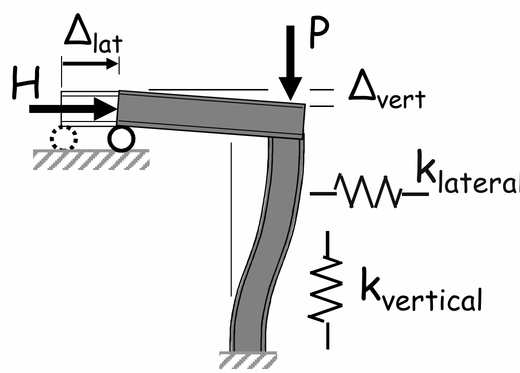
Example

Demonstrate computational method for calculating the elastic critical load (buckling load) for the structural system shown.



Example: Key Stiffness Terms





Vertical Stiffness:

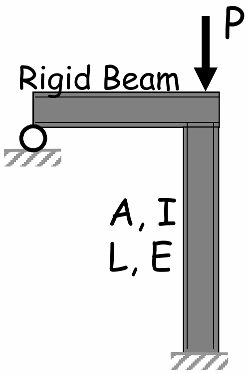
$P = k_{vertical}\,\Delta_{vert}$

Lateral Stiffness:

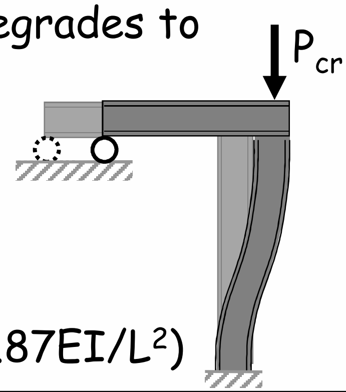
$H = k_{lateral}\,\Delta_{lat}$

$k_{lateral} = 12EI/L^3 - 6P/5L$

Example: Solution



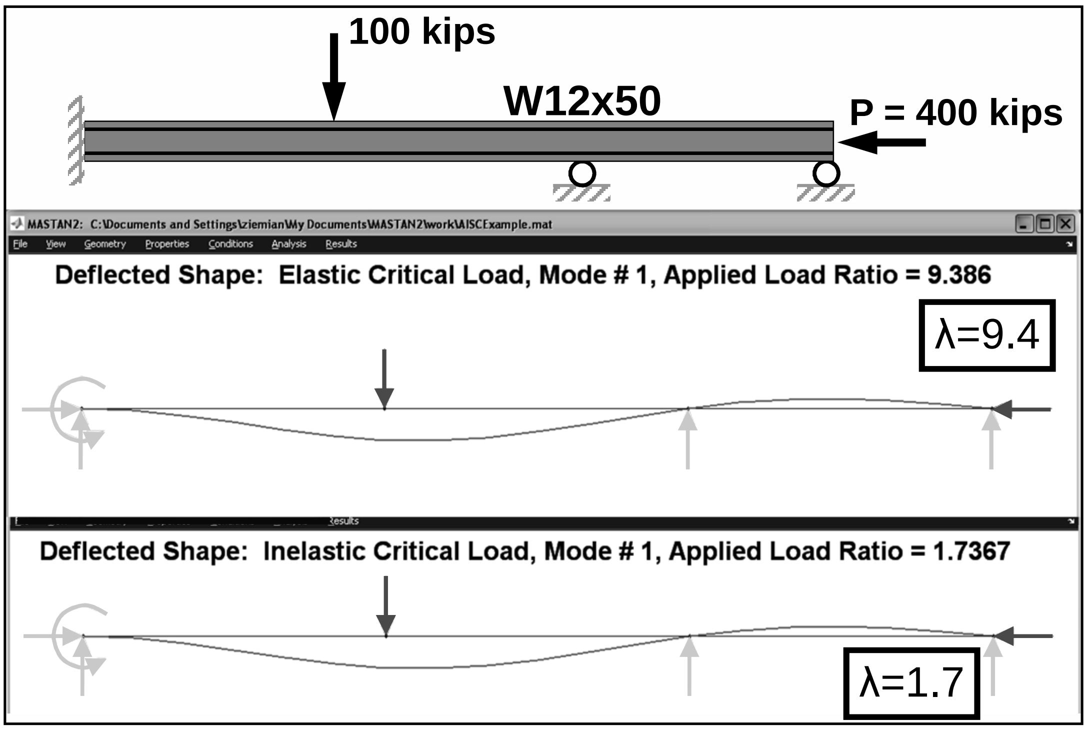
1. Apply reference load, and use 1st-order elastic analysis to obtain internal force distribution.
2. Determine load factor λ at which system stiffness degrades to permit buckling.



$k_{lateral} = 12EI/L^3 - 6\lambda P/5L$

$k_{lateral} = 0$ when $\lambda P = 10EI/L^2$

$P_{cr} = \lambda P = 10EI/L^2$ ($P_{theory}=9.87EI/L^2$)



Thoughts on Critical Load Analysis

- Computer analysis for a large system:
  - First, apply reference and perform analysis
    - Solve equilibrium eqs. $\{F_{ref}\} = [K]\{\Delta\}$
    - With displacements solve for member forces
  - Second, assemble $[K_{el}]$ and $[K_g]$ based on $\{F_{ref}\}$
  - Finally, determine load factor λ causing instability; computationally this means find load factor λ at which $[K]=[K_{el}]+\lambda[K_g]$ becomes singular
    - Determine λ at which $\det([K_{el}]+\lambda[K_g]) = 0$
    - "Eigenvalue" problem: Eigenvalues = Critical Load Factors, λ's
      Eigenvectors = Buckling modes
- Accuracy increases with more elements per compression members (2 often adequate)

Basic Introduction Complete

- Where do I go from here? (Learning to drive)
  - Review the slides (Read the driver's manual)
  - Acquire nonlinear software (Borrow a friend's car)
  - Work lots of examples (Go for a drive, scary at first…)
  - Apply nonlinear analysis in design
    (Formula One? not quite)

Acquire nonlinear analysis software

- Commercial programs
- Educational software
- (i.e. MASTAN2, Strand7, …)

Levels of Analysis:

MASTAN2

1st-Order Elastic: $[K_e]\{\Delta\}=\{F\}$

2nd-Order Elastic: $[K_e + K_g]\{d\Delta\}=\{dF\}$

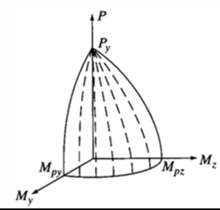
1st-Order Inelastic: $[K_e + K_p]\{d\Delta\}=\{dF\}$

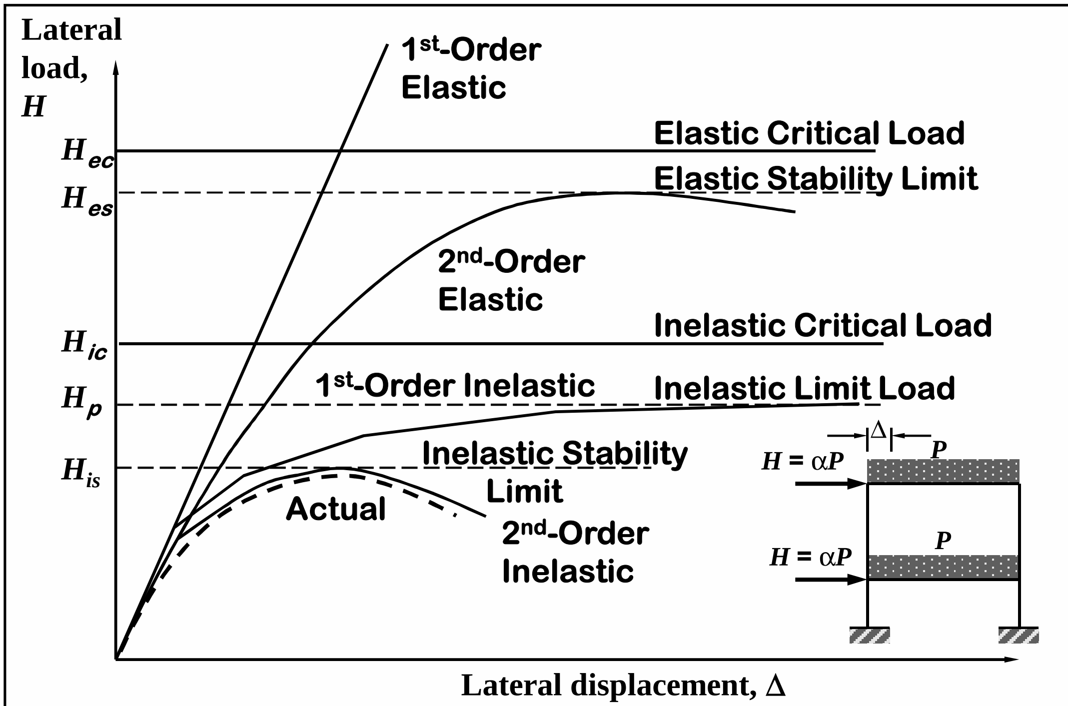
2nd-Order Inelastic: $[K_e + K_g + K_p]\{d\Delta\}=\{dF\}$

Critical Load: $[K_e + \lambda K_g]\{d\Delta\}=\{0\}$

Yield Surface:

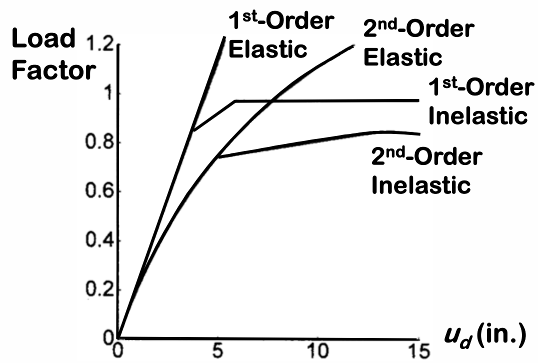
Function of P, M_{major}, and M_{minor}

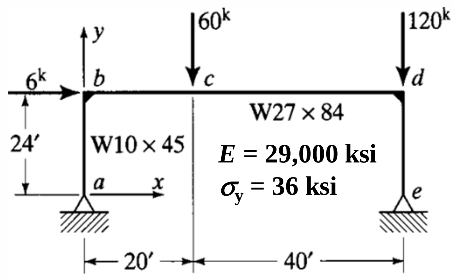


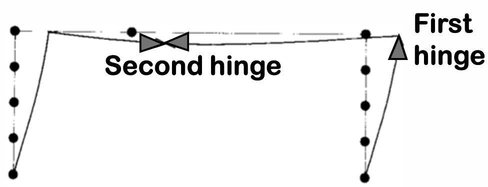


MASTAN2

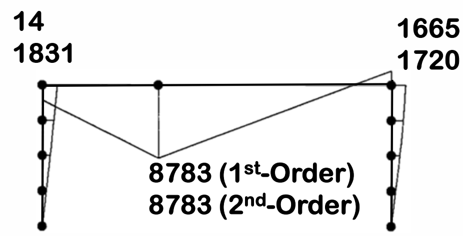
Planar Frame:







Mechanism



Moments at Limit (in. kips)

Summary and Conclusions

- Provided an introduction to nonlinear analysis
  - Review of direct stiffness method
  - Material nonlinear analysis (Inelastic hinge)
  - Geometric nonlinear analysis (2nd-Order)
  - 2nd-Order inelastic analysis (combine above)
  - Critical load analysis ("eigenvalue analysis")
- Nonlinear…think modifying member stiffness!
- All of the above analysis methods appear in and AISC 360, AS4100, and Eurocode 3

References

- Matrix Structural Analysis, 2nd Ed., by McGuire, Gallagher, and Ziemian (Wiley, 2000)
- MASTAN2 at www.mastan2.com
- Tutorial that comes with MASTAN2
- OK, time to jump in and start driving…

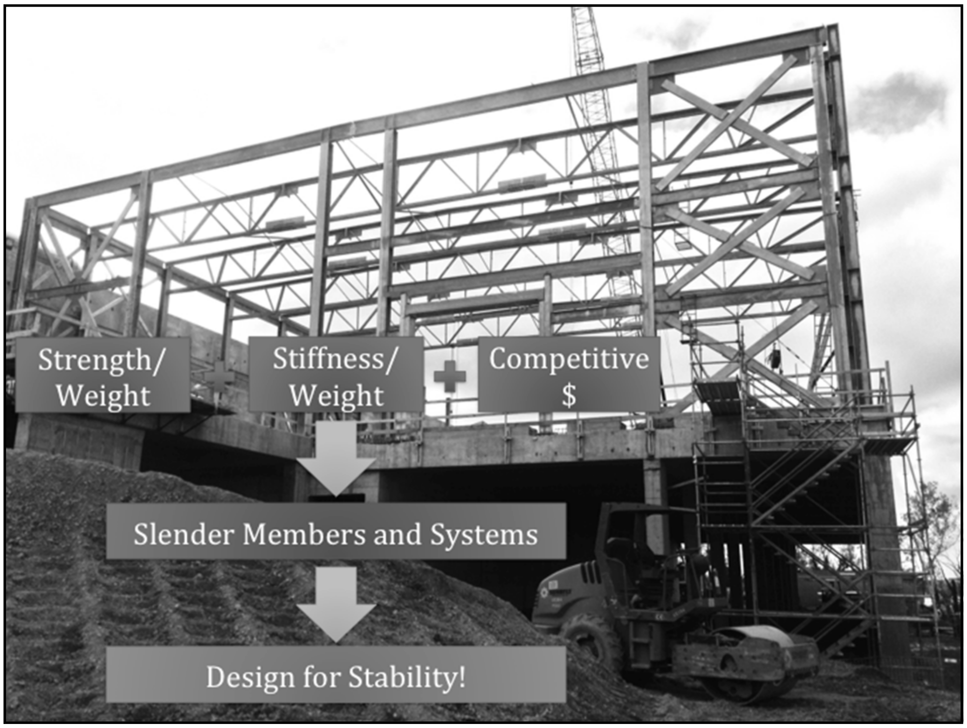
Behavior of Compression Members

Ron Ziemian

Lectures 3 & 4: 13-Aug-2014

Key Definitions

- **Stability:** Under load, component returns to current state after applying a small disturbance such as a deflection
- **Bifurcation (critical load)**: Theoretical point at which loading a component results in an instantaneous change from current state to significant deflection – two options: not buckled or buckled
- **Instability:** Loading a component results in a realistic transition from small deflection to significant deflection – buckling preceded by deflection

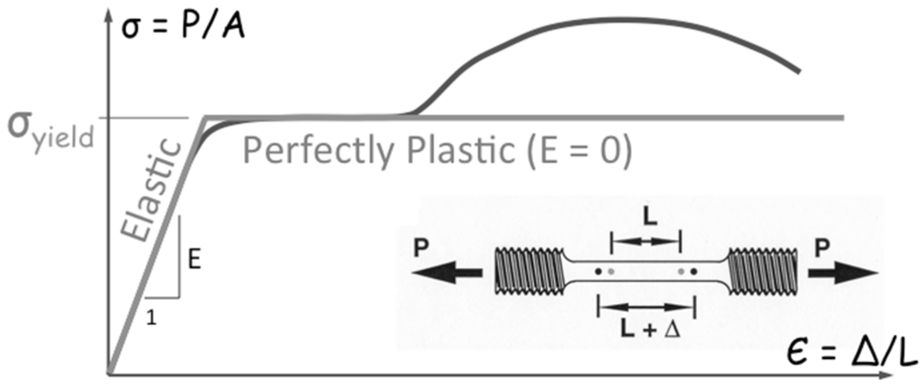


Limit States of Compression Members

- Full yielding (today)
- Instability
  - Along the member length
    - Flexural buckling (today's emphasis!)
    - Torsional buckling
    - Flexural-torsional buckling
  - At the cross section
    - local buckling

Full Yielding

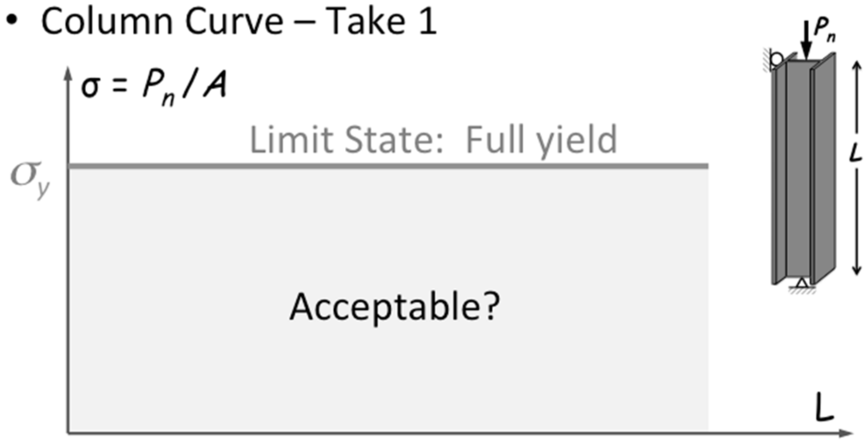
- Tensile test



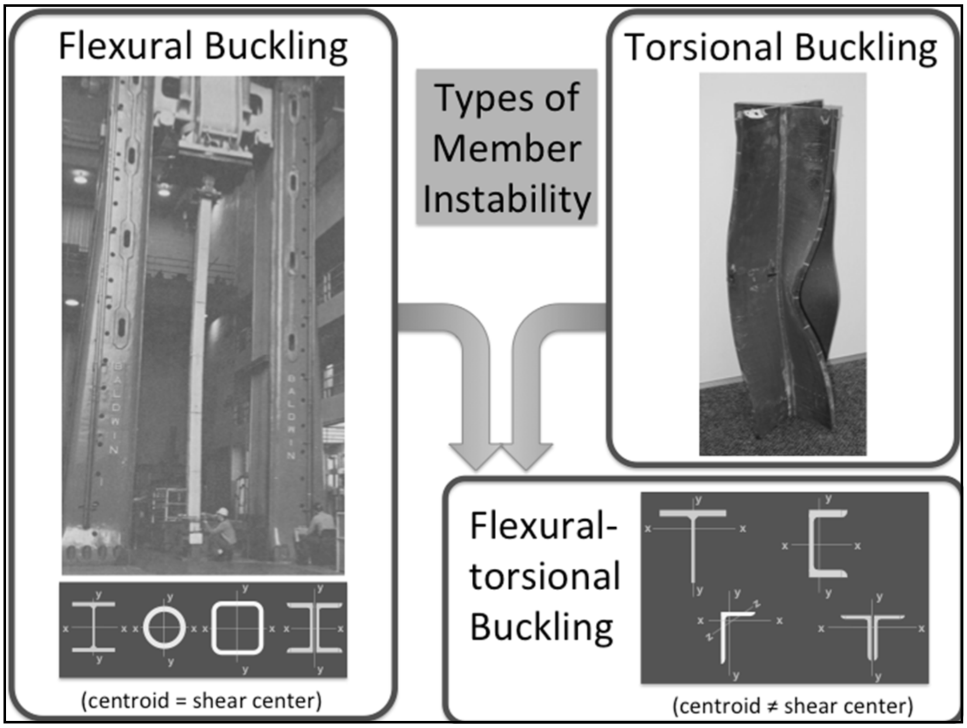
- Assume same response for compression
  - $\sigma_{y,\text{compression}} = \sigma_{y,\text{tension}} = \sigma_{\text{yield}}$
  - Neglect strain hardening (assume elastic-plastic)

Full Yielding (2)

- Column Curve – Take 1



- What about:
  - member instability ??? (tonight!)
  - cross section instability (local buckling) ???



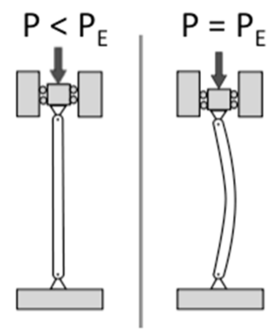
Flexural Buckling

- Euler's column
  - solution
  - assumptions
- Undoing Euler's assumptions (approaching reality)
  - bending before bifurcation
  - not fully elastic (partial yielding)
  - support conditions
- Column curves
  - AISC, AS4100, Eurocode 3, ...
  - others

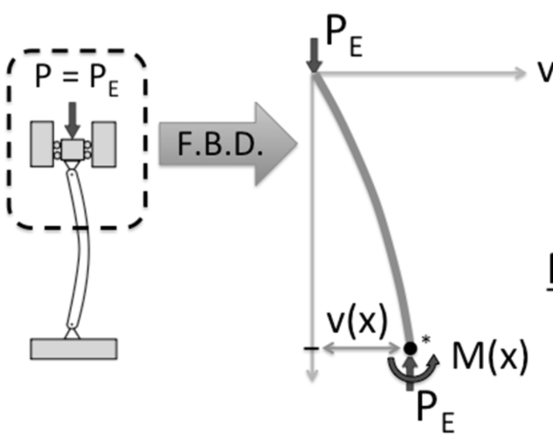
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)


METHODUS
INVENIENDI
LINEAS CURVAS
Maximi Minimive proprietate gaudentes,
SIVE
SOLUTIO
PROBLEMATIS ISOPERIMETRICI
LATISSIMO SENSU ACCEPTI
AUCTORE
LEONHARDO EULERO,
Professore Regio, & Academiae Imperialis Scientiarum Petropolitanae Socio.
LAUSANNÆ & GENEVÆ,
Apud MARCUM-MICHAELEM BOUSQUET & Socios.
MDCCXLIV.



Euler Buckling (2)



<u>Equilibrium</u>:

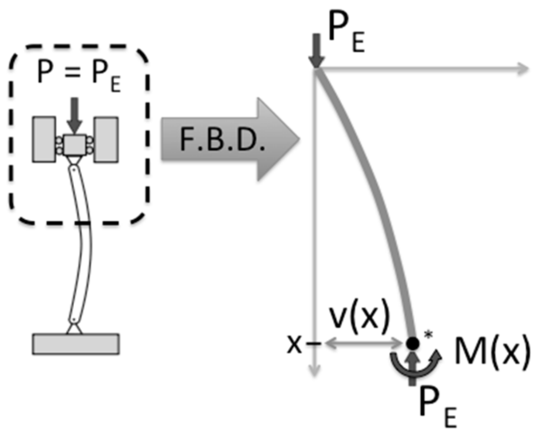
$$\Sigma M_* = 0$$

$$M(x) + P_E v(x) = 0$$

<u>Moment-curvature</u>:

$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

Euler Buckling (3)



<u>Equilibrium</u>:

$$\Sigma M_* = 0$$

$$M(x) + P_E v(x) = 0$$

<u>Moment-curvature</u>:

$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

$$EI\frac{d^2v}{dx^2} + P_E v = 0 \quad \Rightarrow$$

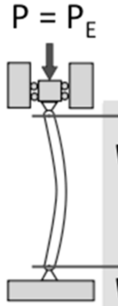
Solution:

$$v(x) = C_1\cos\left(\sqrt{\frac{P_E}{EI}}x\right) + C_2\sin\left(\sqrt{\frac{P_E}{EI}}x\right)$$

wolframalpha.com
a2*y''(x)+a1*y(x)=0

Euler Buckling (4)

$$v(x) = C_1\cos\left(\sqrt{\frac{P_E}{EI}}x\right) + C_2\sin\left(\sqrt{\frac{P_E}{EI}}x\right)$$



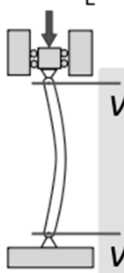
$$v(x=0) = 0 \quad \Rightarrow \quad C_1 = 0 \quad \Rightarrow \quad v(x) = C_2\sin\left(\sqrt{\frac{P_E}{EI}}x\right)$$

Boundary Conditions!

$$v(x=L) = 0$$

Euler Buckling (5)

P = P_E



$$v(x=0)=0 \Rightarrow v(x)=C_2\sin\left(\sqrt{\frac{P_E}{EI}}x\right)$$

Boundary Conditions!

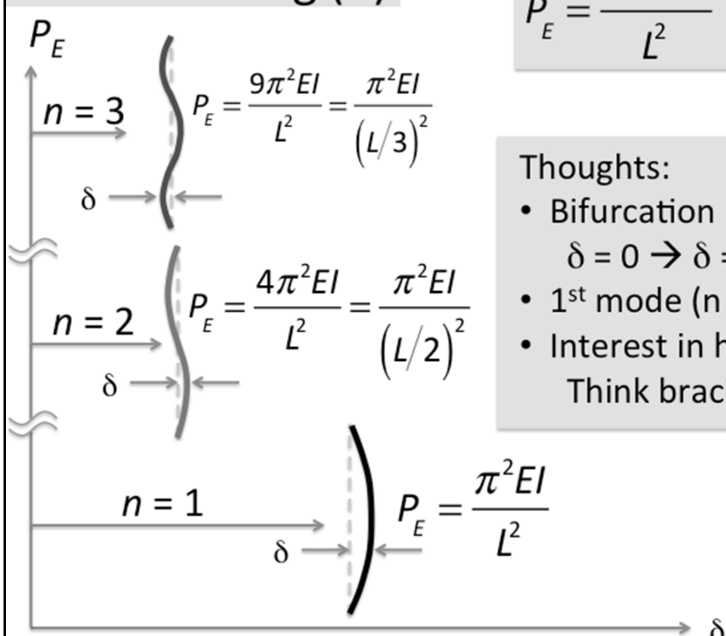
$$v(x=L)=0 \Rightarrow v(x=L)=0=C_2\sin\left(\sqrt{\frac{P_E}{EI}}L\right)$$

1) $C_2=0$ "trivial solution"

2) $\sin\left(\sqrt{\frac{P_E}{EI}}L\right)=0 \Rightarrow \sqrt{\frac{P_E}{EI}}L=n\pi \Rightarrow$ $\boxed{P_E=\frac{n^2\pi^2EI}{L^2} \quad n=1,2,3,\ldots}$

Euler Buckling (6)

$$P_E=\frac{n^2\pi^2EI}{L^2} \quad n=1,2,3,\ldots$$



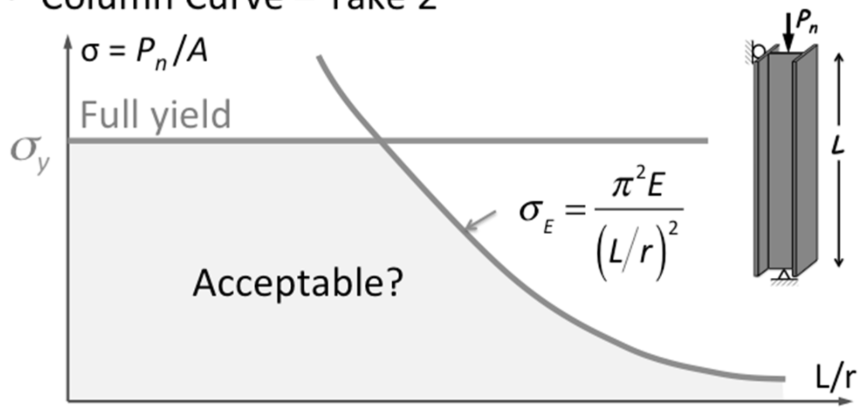
Thoughts:
- Bifurcation
  δ = 0 → δ = unbounded
- 1st mode (n = 1) controls!
- Interest in higher modes?
  Think bracing!

Euler Buckling (7)

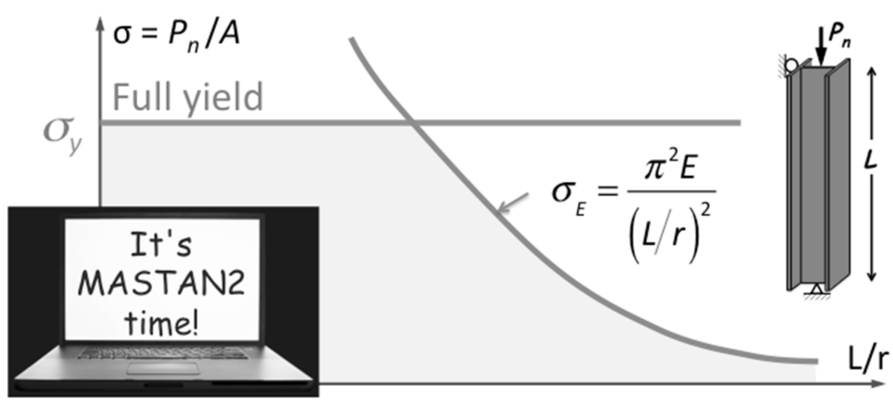
- Euler Buckling Stress

$$P_E = \frac{\pi^2 EI}{L^2} \;\Rightarrow\; \sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{\left(L/r\right)^2} \quad \text{with } r = \sqrt{\frac{I}{A}}$$

- Column Curve – Take 2



- What about those assumptions?

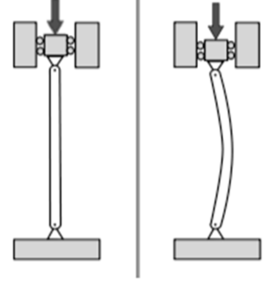


W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{cr,MNA}$	$P_{cr,GNA}$	$P_{cr,min}$
50	199			
150	597			

Euler Buckling

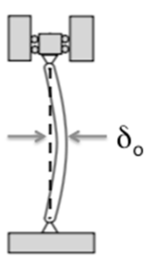
- Leonhard Euler, 1744 and 1757
- Assumptions
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)



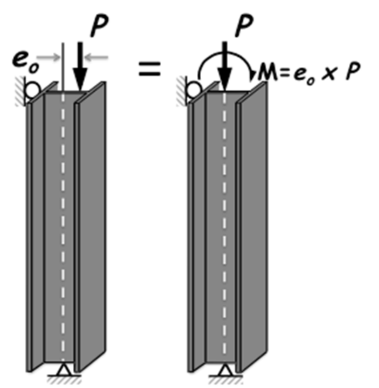
Bending

- Bending can be produced by:

1. Prior to loading, column is not perfectly straight



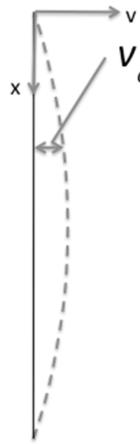
2. Axial load not concentrically applied (e_o is small, but not zero!)



Reality: Some combination of above exists…

Bending (2)

Let’s consider a column with initial out-of-straightness:

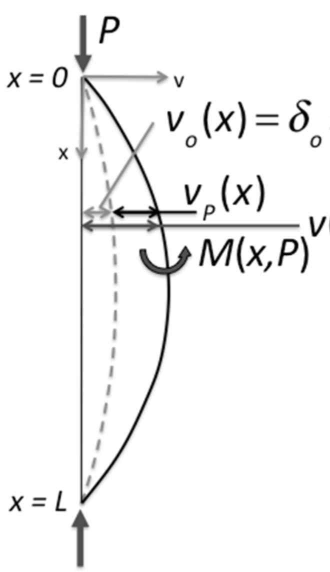


$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

Initial imperfection at mid-length
e.g. δ_o = L/1000

Bending (3)

Column with initial out-of-straightness:

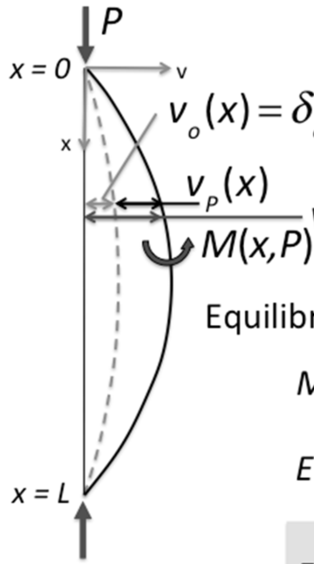


$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v(x) = v_o(x) + v_P(x)$$

Bending (4)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v(x) = v_o(x) + v_P(x)$$

Equilibrium → Differential Equation:

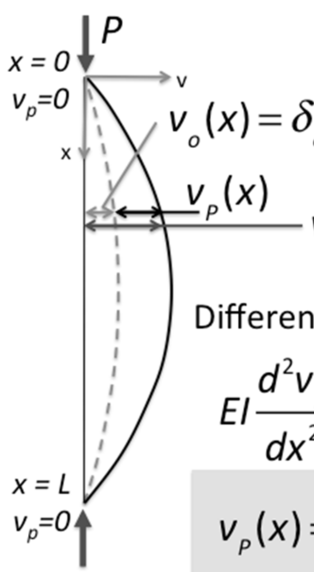
$$M(x,P) + Pv(x) = 0$$

$$EI\frac{d^2 v_P}{dx^2} + P\left(v_o(x) + v_P(x)\right) = 0$$

$$EI\frac{d^2 v_P}{dx^2} + Pv_P(x) = -Pv_o(x) = -P\delta_o \sin\frac{\pi x}{L}$$

Bending (5)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

wolframalpha.com
a2*y''(x)+a1*y(x)=-a1*a3*sin(a4*x)

$$v(x) = v_o(x) + v_P(x)$$

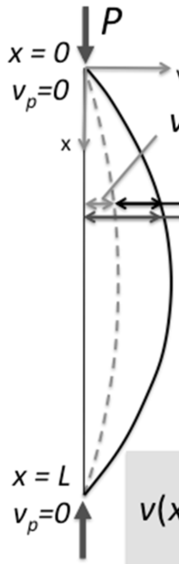
Differential Equation → Solution with BC's

$$EI\frac{d^2 v_P}{dx^2} + Pv_P(x) = -P\delta_o \sin\frac{\pi x}{L}$$

$$v_P(x) = \frac{1}{\frac{EI\pi^2}{PL^2} - 1}\delta_o \sin\frac{\pi x}{L} = \frac{1}{\frac{P_E}{P} - 1}\delta_o \sin\frac{\pi x}{L}$$

Bending (6)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v_P(x)$$

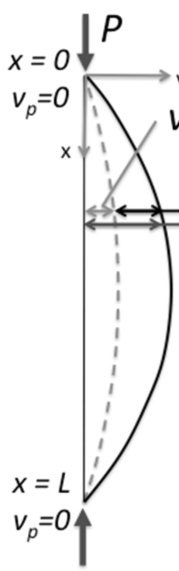
$$v(x) = v_o(x) + v_P(x)$$

$$v_P(x) = \frac{1}{\frac{P_E}{P} - 1}\delta_o \sin\frac{\pi x}{L}$$

$$v(x) = \delta_o \sin\frac{\pi x}{L} + \frac{1}{\frac{P_E}{P} - 1}\delta_o \sin\frac{\pi x}{L} = (1 + \frac{1}{\frac{P_E}{P} - 1})\delta_o \sin\frac{\pi x}{L}$$

Bending (7)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v_P(x)$$

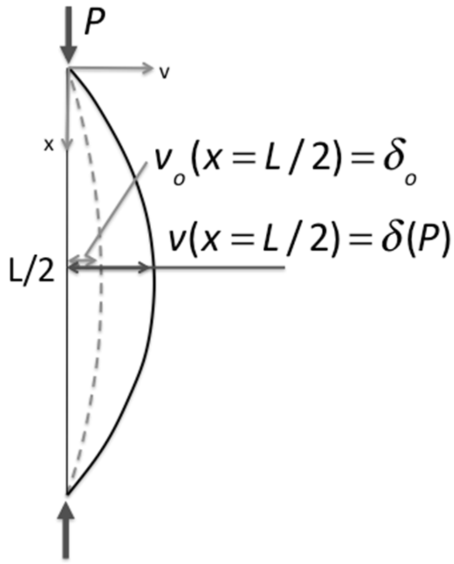
$$v(x) = v_o(x) + v_P(x)$$

$$v(x) = (1 + \frac{1}{\frac{P_E}{P} - 1})\delta_o \sin\frac{\pi x}{L}$$

$$v(x) = \frac{1}{1 - \frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L} \Rightarrow v(x) = \frac{1}{1 - \frac{P}{P_E}} v_o(x)$$

Bending (8)

Column with initial out-of-straightness:

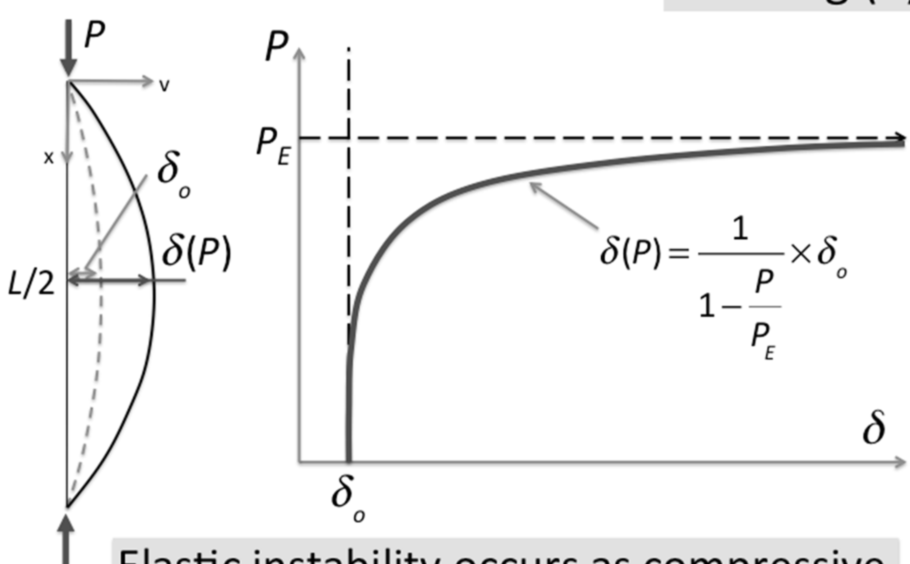


$$v(x) = \frac{1}{1 - \frac{P}{P_E}} v_o(x)$$

$$\delta(P) = \frac{1}{1 - \frac{P}{P_E}} \times \delta_o$$

Bending (9)

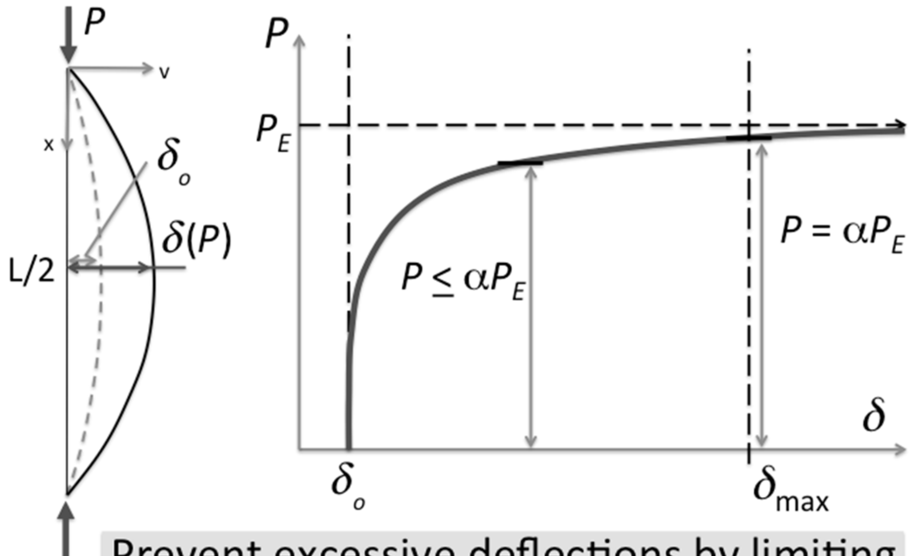
Column with initial out-of-straightness:



Elastic instability occurs as compressive force P approaches Euler critical load P_E

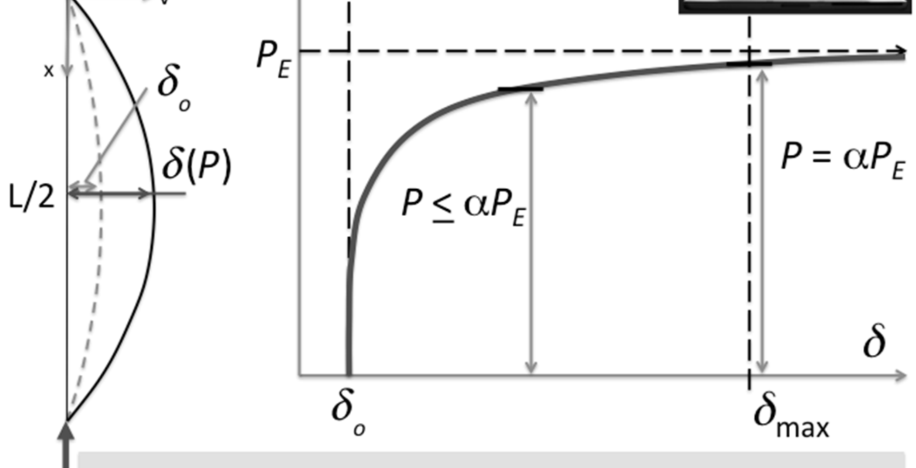
Column with initial out-of-straightness:

Bending (10)



Prevent excessive deflections by limiting P to some proportion of P_E, i.e. $P \le \alpha P_E$

Column with initial out-of-straightness:



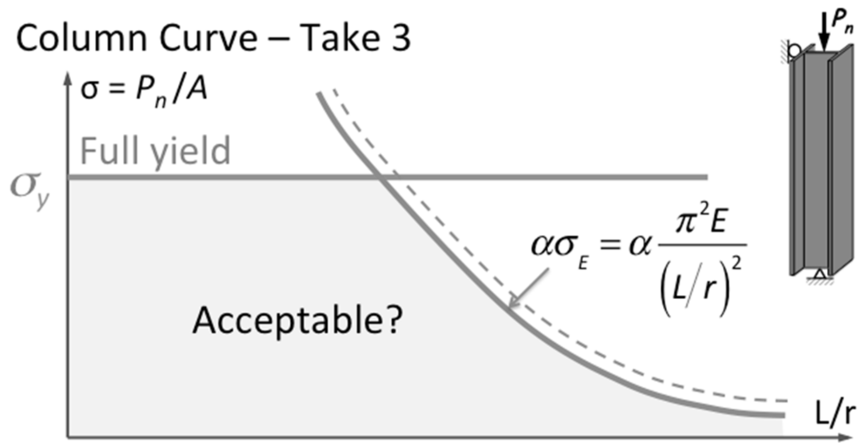
W14x145 (E=29,000, F_y = 50, I_{minor}=1710)
L = 597 and δ_o = L/1000 = 0.0.597 **GNIA**

Bending (11)

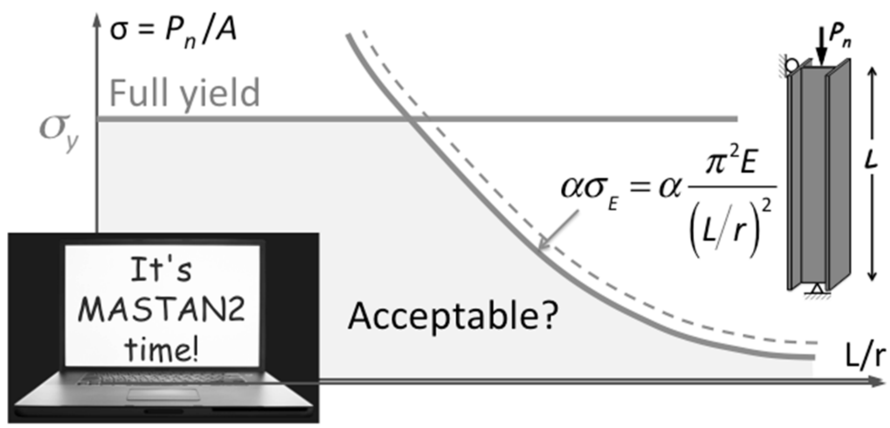
- Limit elastic bending deflections

$$P \le \alpha P_E \;\Rightarrow\; \frac{P}{A} \le \alpha \frac{P_E}{A} \;\Rightarrow\; \sigma \le \alpha \sigma_E$$

- Column Curve – Take 3



- Consider yielding due to bending plus axial force?

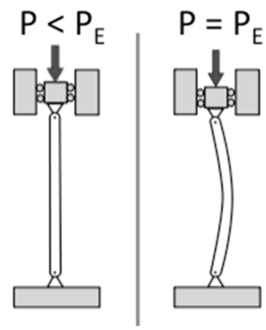


W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{cr,GNA}$	$P_{cr,GNIA}$	$P_{cr,min}$
150	597			

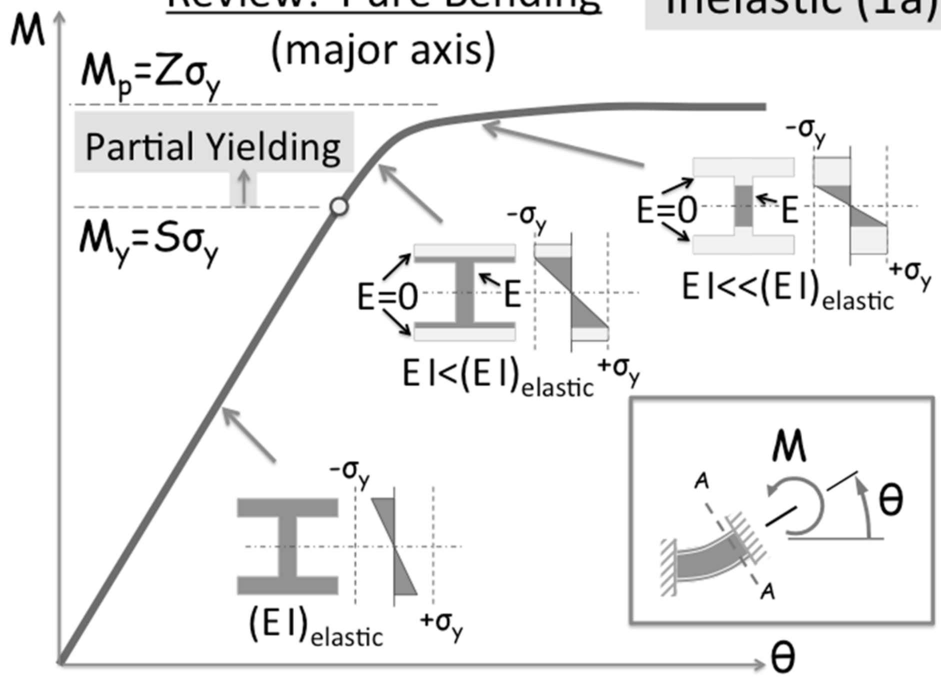
Euler Buckling

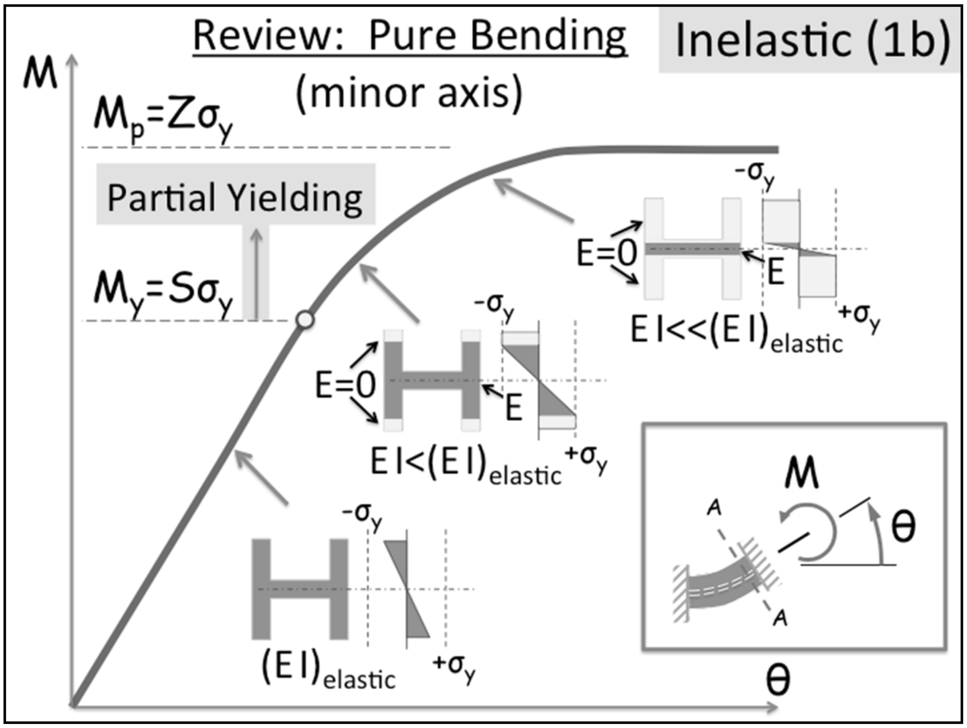
- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)



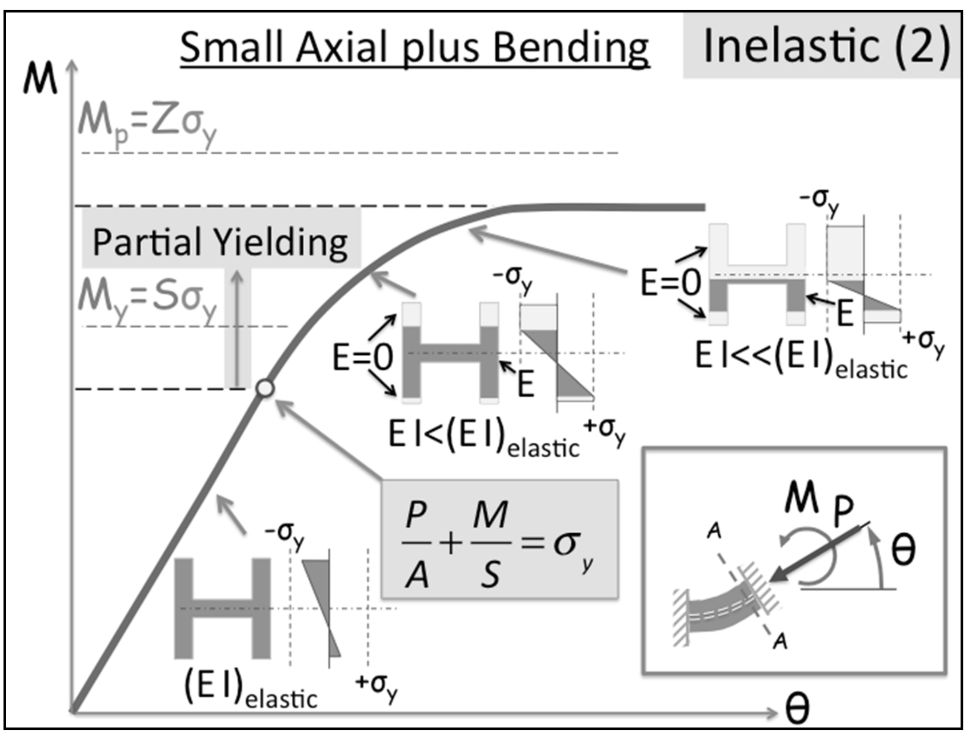
Review: Pure Bending (major axis)

Inelastic (1a)

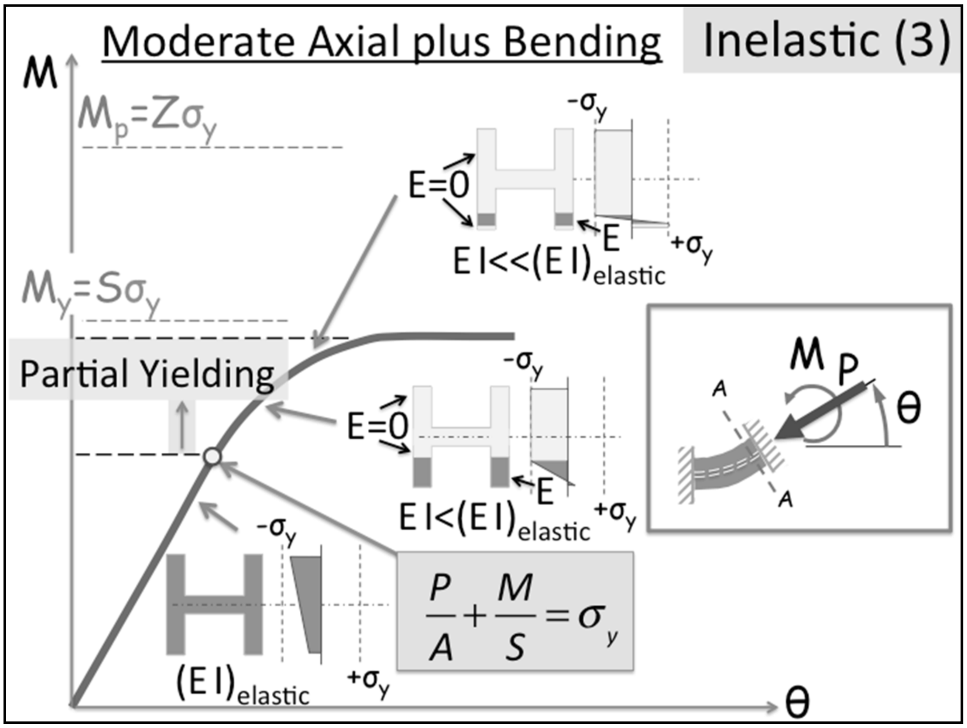


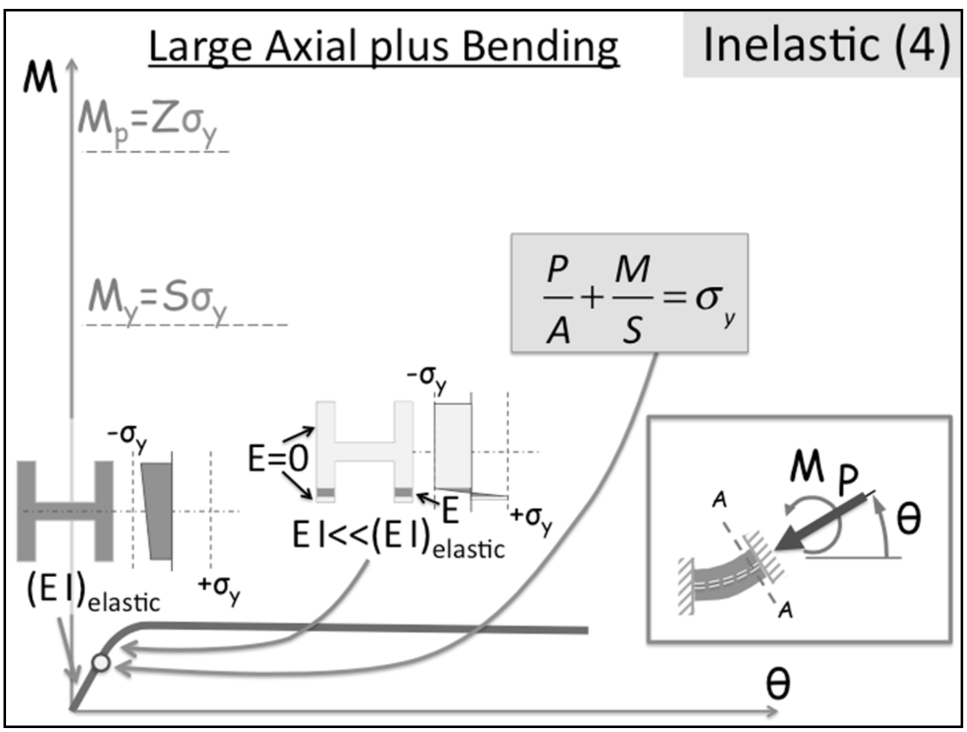


Review: Pure Bending
(minor axis)
Inelastic (1b)
M
$M_p=Z\sigma_y$
Partial Yielding
$M_y=S\sigma_y$
$-\sigma_y$
E=0
E
EI<<(EI)elastic
$+\sigma_y$
EI<(EI)elastic
(EI)elastic
A
θ



Small Axial plus Bending
Inelastic (2)
M
$M_p=Z\sigma_y$
Partial Yielding
$M_y=S\sigma_y$
$-\sigma_y$
E=0
E
EI<<(EI)elastic
$+\sigma_y$
EI<(EI)elastic
$\frac{P}{A}+\frac{M}{S}=\sigma_y$
(EI)elastic
M P
A
θ

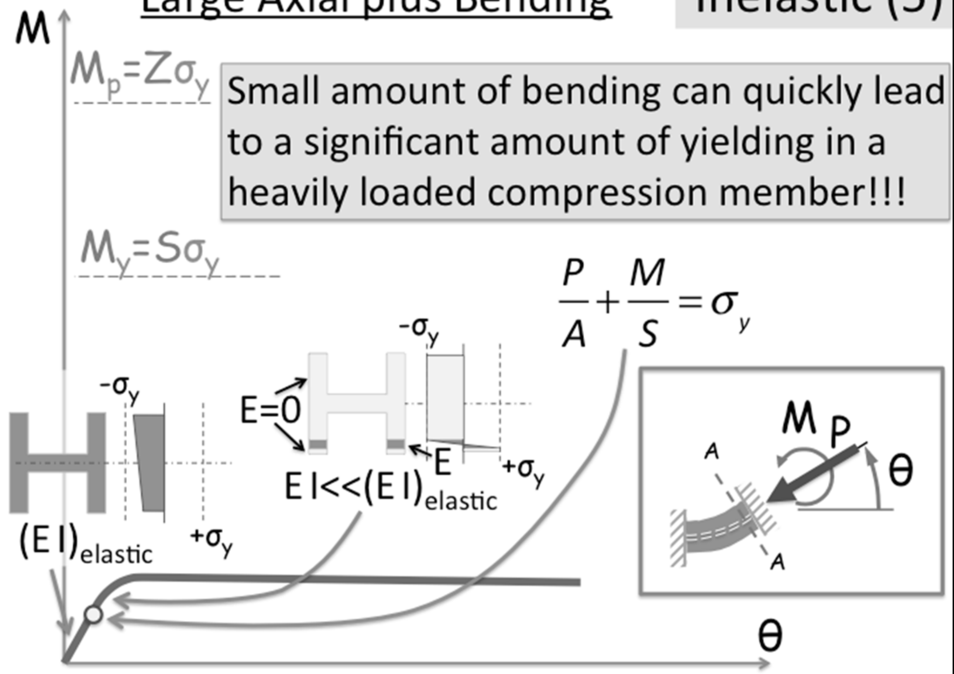

Moderate Axial plus Bending
Inelastic (3)
M
Mp=Zσy
My=Sσy
E=0
E
EI<<(EI)elastic
-σy
+σy
Partial Yielding
EI<(EI)elastic
(EI)elastic
P/A + M/S = σy
M
P
θ
A
θ


Large Axial plus Bending
Inelastic (4)
M
Mp=Zσy
My=Sσy
P/A + M/S = σy
-σy
+σy
E=0
E
EI<<(EI)elastic
(EI)elastic
M
P
θ
A
θ

Large Axial plus Bending

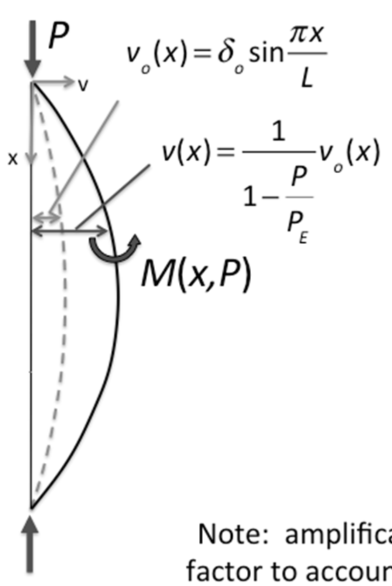
Inelastic (5)

Small amount of bending can quickly lead to a significant amount of yielding in a heavily loaded compression member!!!



Closer look at that bending:

Inelastic (6)



Equilibrium:

$$M(x,P) + Pv(x) = 0$$

$$M(x,P) = -Pv(x)$$

$$M(x,P) = -P\frac{1}{1-\frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L}$$

$$M(x,P) = \frac{1}{1-\frac{P}{P_E}} \cdot \left(-P\delta_o \sin\frac{\pi x}{L}\right)$$

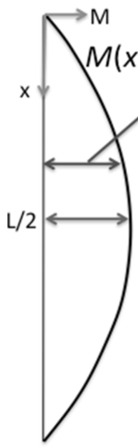
Note: amplification factor to account for 2^{nd}-order effects

$$\boxed{M(x,P) = \frac{1}{1-\frac{P}{P_E}} \cdot M(x,P)^{1st-order}}$$

Inelastic (7)

Closer look at that bending:

<u>Elastic M-diagram:</u>



$$M(x,P) = \frac{-P}{1-\frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L}$$

$$M(\frac{L}{2},P) = \frac{-P}{1-\frac{P}{P_E}}\delta_o$$

All is good…as long as all is elastic, i.e. no yielding!

$$\left|\frac{P}{A}\right| + \left|\frac{M(x,P)}{S}\right| < \sigma_y$$

But, yielding will occur when

$$\left|\frac{P}{A}\right| + \left|\frac{M(L/2,P)}{S}\right| = \sigma_y$$

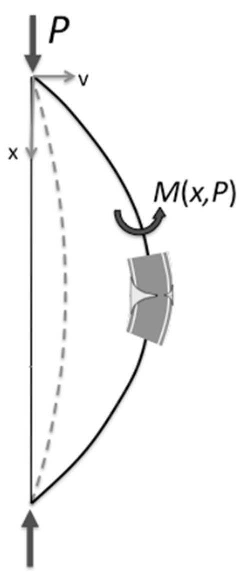
or, an axial load P that satisfies:

$$\frac{P}{A} + \frac{1}{\left(1-\frac{P}{P_E}\right)}\frac{P\delta_o}{S} = \sigma_y$$

Note: relatively simple equation to compute axial force that produces first yield (excludes σ_{res})

Inelastic (8)

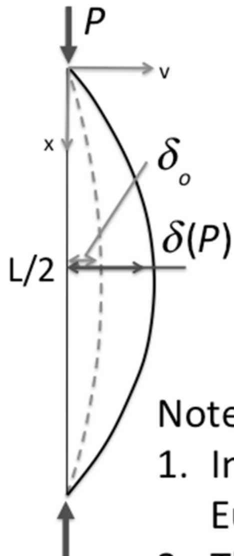
And, once yielding occurs (ouch!):

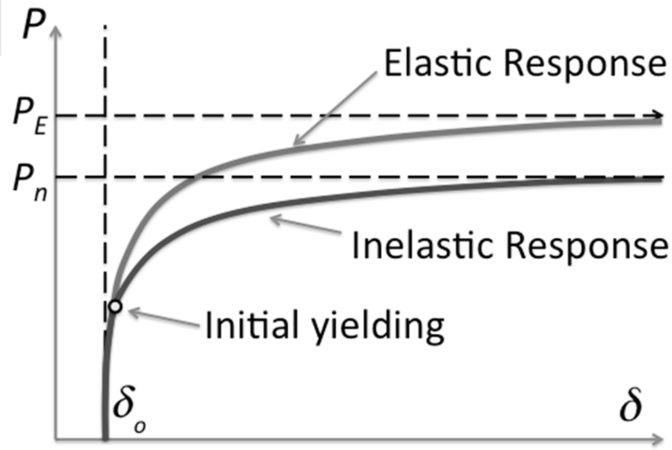


1. Yielded portion loses stiffness, $EI \downarrow$
2. Increases in deflection, $v(x) \uparrow$
3. Increases moment, $M(x) = P \cdot v(x) \uparrow$
4. Resulting in more yielding…

5. If equilibrium, apply more P
6. Repeat above steps 1 to 4
7. Apply more P repeating steps 1 to 6 until instability!

Inelastic (9)





Notes:

1. Inelastic instability occurs below the Euler critical load, i.e. $P_n < P_E$
2. The smaller the column slenderness L/r, the further P_n is below P_E $(L/r \downarrow \ P_E \Uparrow)$

Inelastic (10)

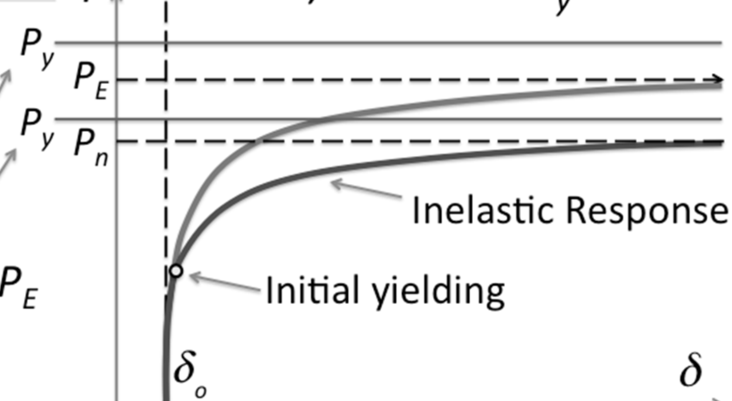
But, where is P_y?

Only two options:

$P_y \geq P_E$

-or-

$P_n \leq P_y < P_E$



Notes:

1. Because of bending, P_n cannot exceed P_y
2. The larger the column slenderness L/r, the more bending, and the further P_n is below P_y

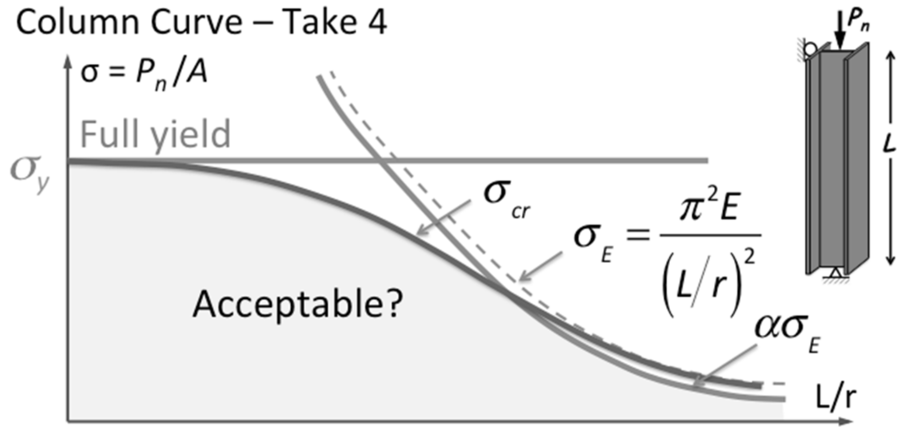
Inelastic (11)

- Axial plus bending may cause yielding

$$\sigma_{cr} = \frac{P_n}{A} \qquad L/r \rightarrow 0,\ \sigma_{cr} = \sigma_y$$

$$L/r \uparrow,\ \sigma_{cr} < \sigma_y \text{ and } \sigma_{cr} < \sigma_E$$

- Column Curve – Take 4

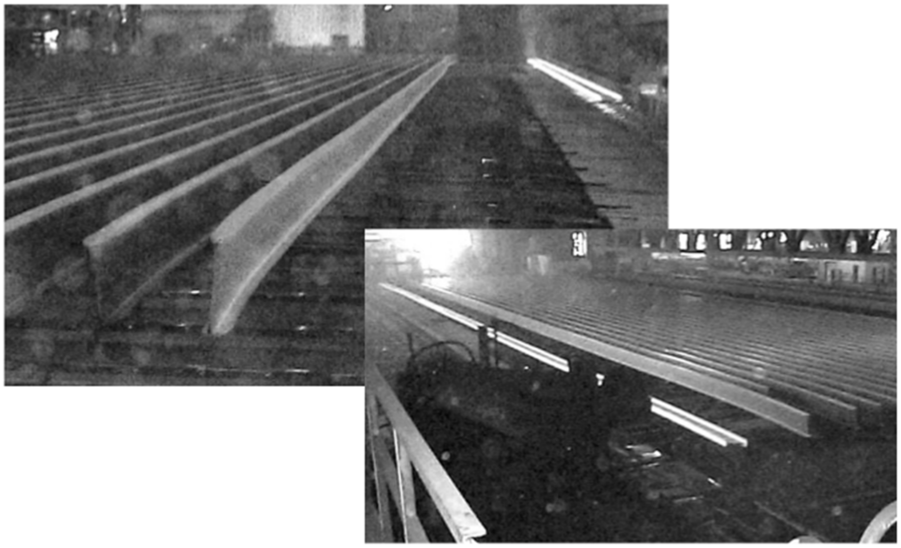


- What about residual stresses?

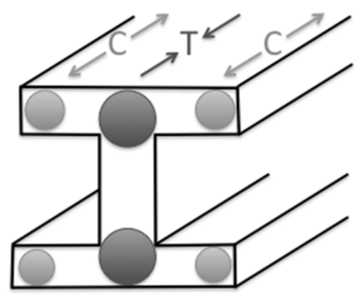
Residual Stresses

- Occurs in structural shapes
  - Uneven cooling of hot-rolled shape after rolling
  - Welding of plates for fabricated or built-up shapes
  - Cold bending during fabrication
- Magnitude and distribution of residual stresses depend on the cross-sectional shape and dimensions
- Residual stresses are usually independent of steel yield strength
- Thermal residual stresses occur in rolled wide flange shapes because locations with high surface area (e.g., flange tips) cool well before locations with smaller surface area (flange-to-web intersections)

Residual Stresses (2)



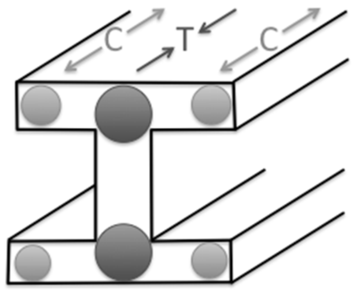
Residual Stresses (3)

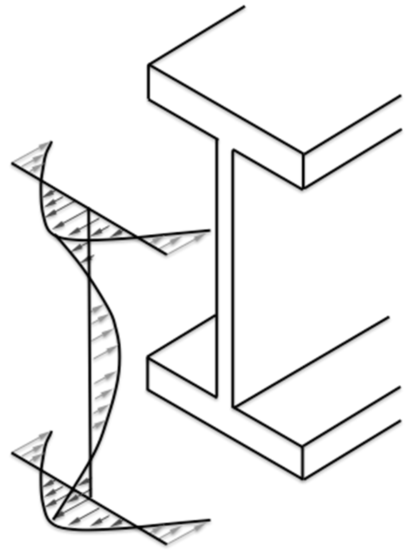


1. Entire section hot and starts to cool…lengthwise contraction with E_o << E
2. Flange tips (surface area!) cool relatively faster than flange-web intersection (smaller surface) area, $E_{fl} \approx E$
3. Flange-web intersection (smaller surface area) now cools and wants to contract, but flange tips are already set and do not want to contract.
4. Result – locations to cool last end up in tension and equilibrium requires locations that cooled first to end up in compression.

Residual Stresses (4)

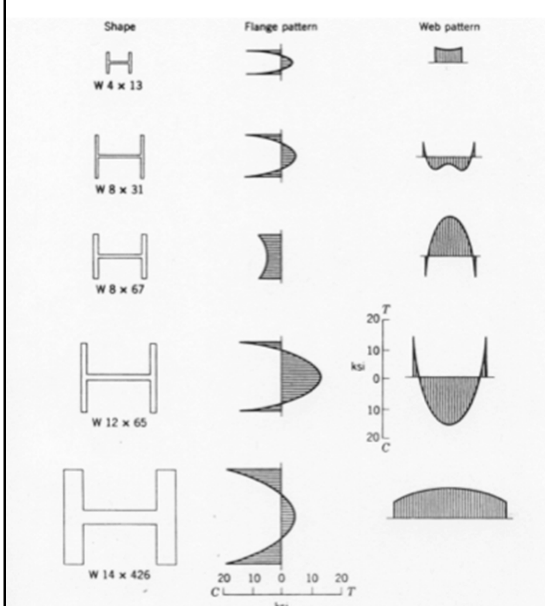
From previous slide

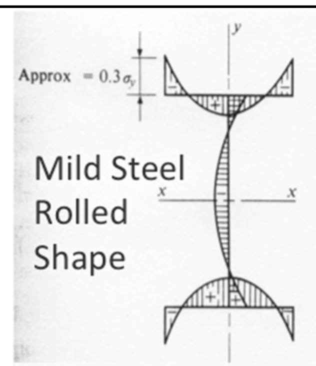


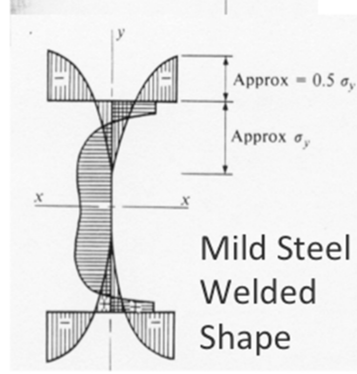


Closer to actual distribution

Residual Stresses (5)



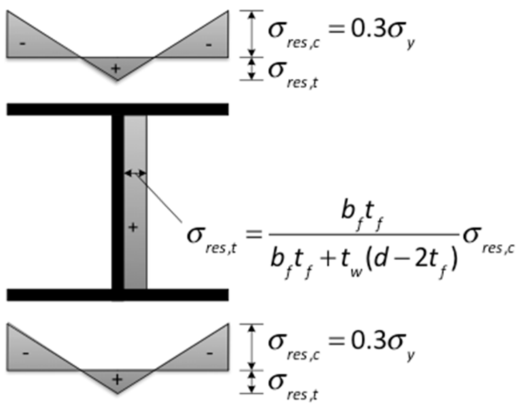




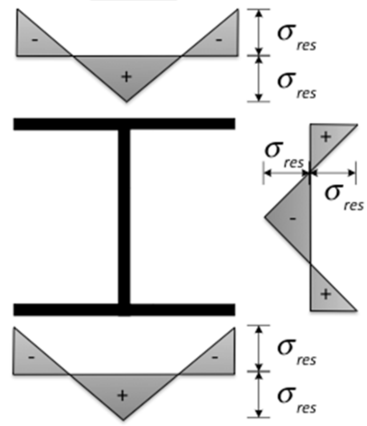
Residual Stresses (6)

Residual Stresses patterns often used in computational studies:

Galambos and Ketter



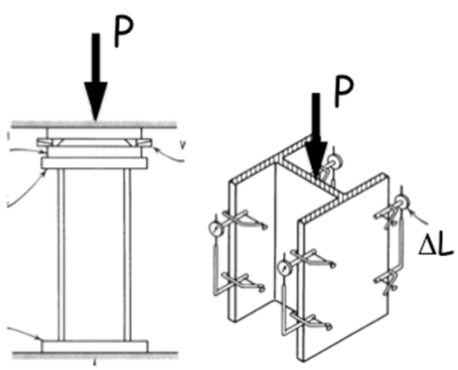
ECCS

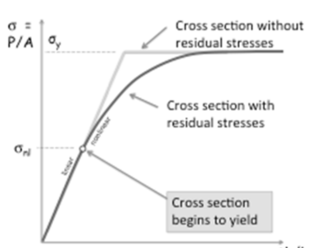


$$d/b_f \le 1.2 \;\Rightarrow\; \sigma_{res} = 0.5\sigma_y$$
$$d/b_f > 1.2 \;\Rightarrow\; \sigma_{res} = 0.3\sigma_y$$

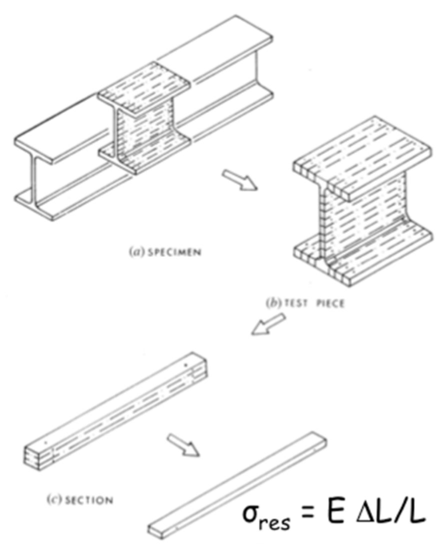
Residual Stresses (7)

Stub Column Test

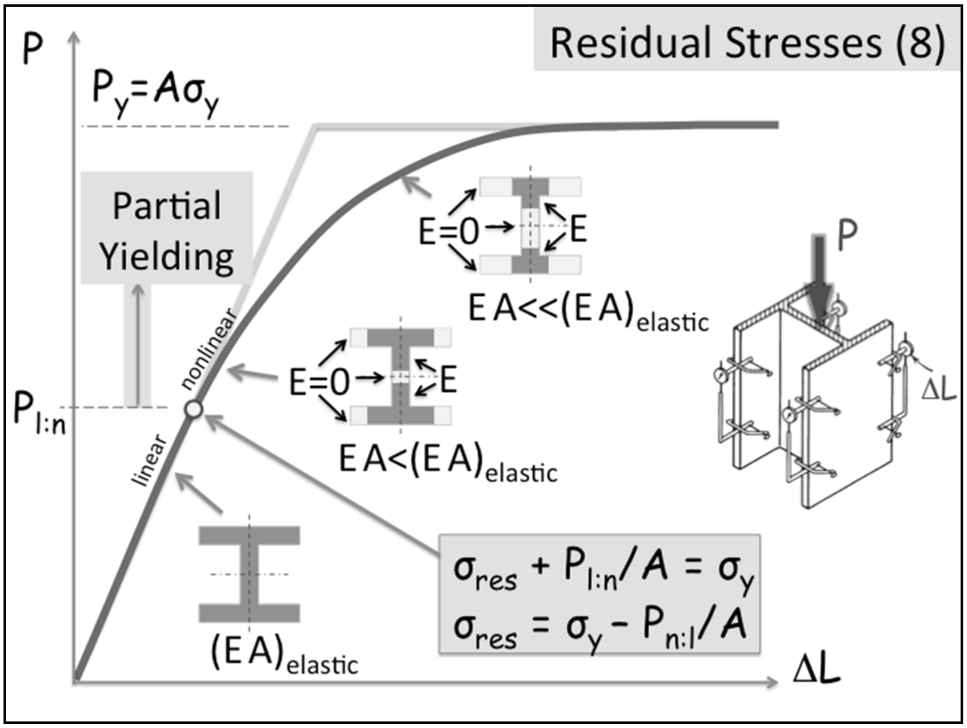


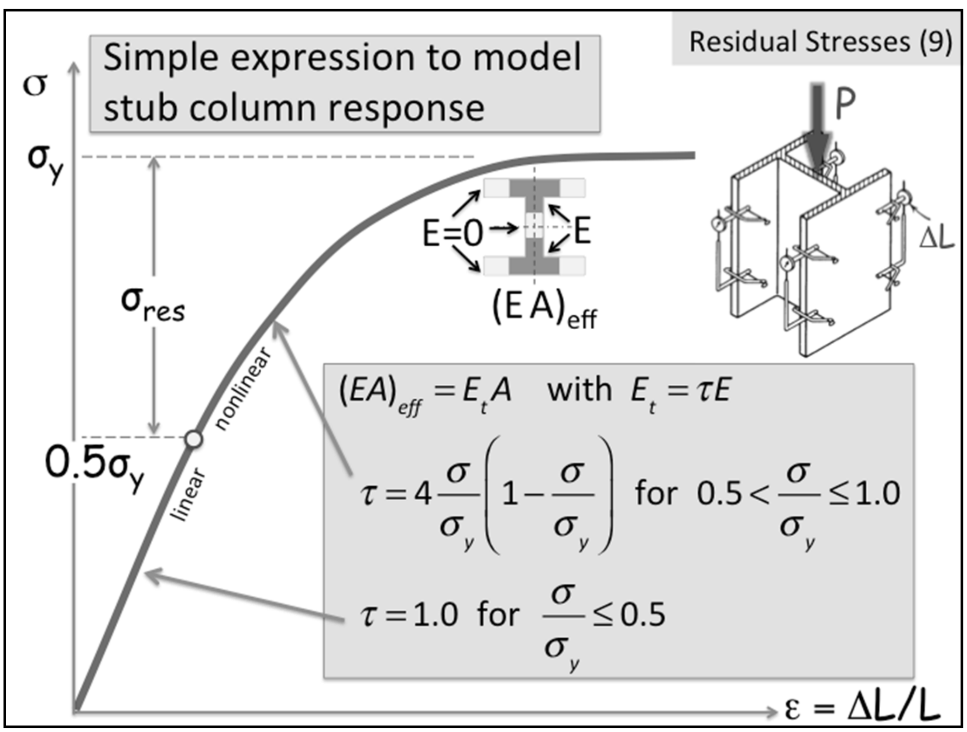


Sectioning



$\sigma_{res} = E\ \Delta L/L$


Residual Stresses (8)
P
P_y=Aσ_y
Partial Yielding
nonlinear
linear
P_l:n
E=0
E
EA<<(EA)_elastic
EA<(EA)_elastic
(EA)_elastic
P
ΔL
σ_res + P_l:n/A = σ_y
σ_res = σ_y – P_n:l/A
ΔL


Residual Stresses (9)
Simple expression to model stub column response
σ
σ_y
σ_res
0.5σ_y
nonlinear
linear
E=0
E
(EA)_eff
P
ΔL
(EA)_eff = E_t A with E_t = τE
τ = 4 σ/σ_y (1 − σ/σ_y) for 0.5 < σ/σ_y ≤ 1.0
τ = 1.0 for σ/σ_y ≤ 0.5
ε = ΔL/L

Residual Stresses (10)

- Euler -to- Inelastic Buckling Stress

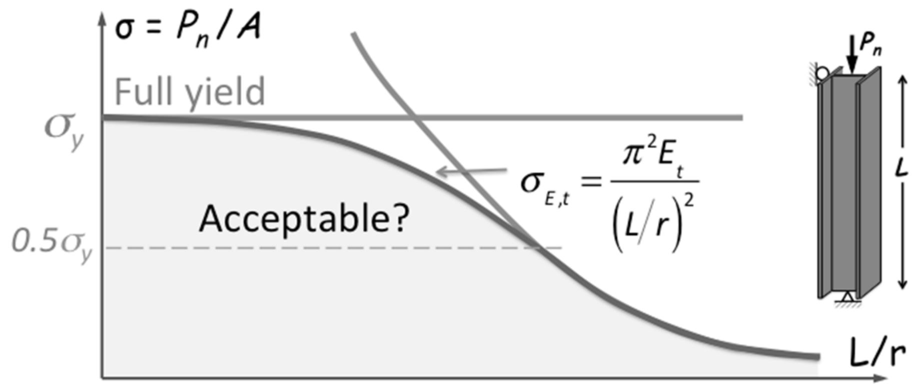
$$\sigma_E = \frac{\pi^2 E}{\left(L/r\right)^2} \Rightarrow \sigma_{E,t} = \frac{\pi^2 E_t}{\left(L/r\right)^2}$$

$E_t = \tau E$

$\tau = 4\frac{\sigma}{\sigma_y}\left(1-\frac{\sigma}{\sigma_y}\right)$ for $0.5 < \frac{\sigma}{\sigma_y} \le 1.0$

$\tau = 1.0$ for $\frac{\sigma}{\sigma_y} \le 0.5$

- Column Curve – Take 5



- But wait! What about bending?

Residual Stresses (11)

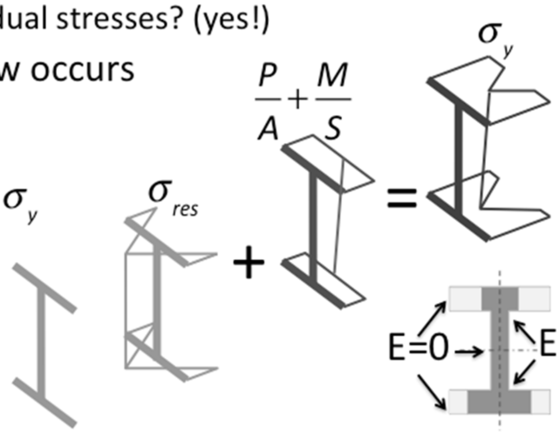
- Compression members include
  - Bending without residual stresses? (no!)
  - No bending with residual stress? (no!)
  - Bending with residual stresses? (yes!)

<u>1950-70's</u>
British Standard
AISC

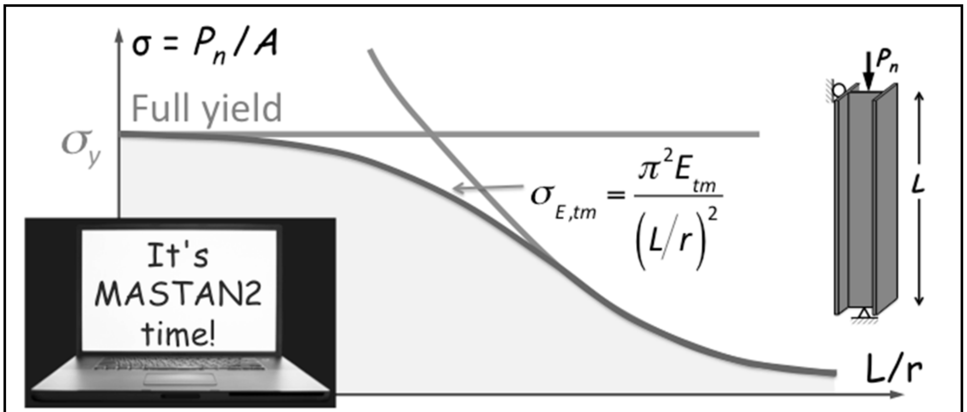
- Partial yielding now occurs sooner when:

$$\sigma_{res} + \left(\frac{P}{A} + \frac{M}{S}\right) = \sigma_y$$

Note: *M* is due to initial imperfection and/or non-concentric loading



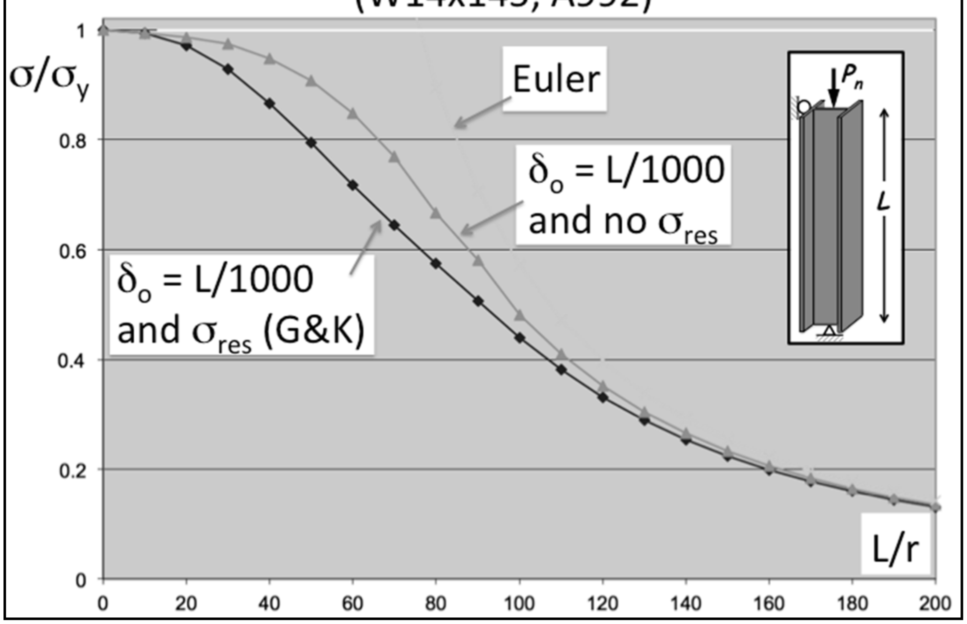
- Partial yielding = loss of flexural stiffness, $EI \ll EI_{elastic}$

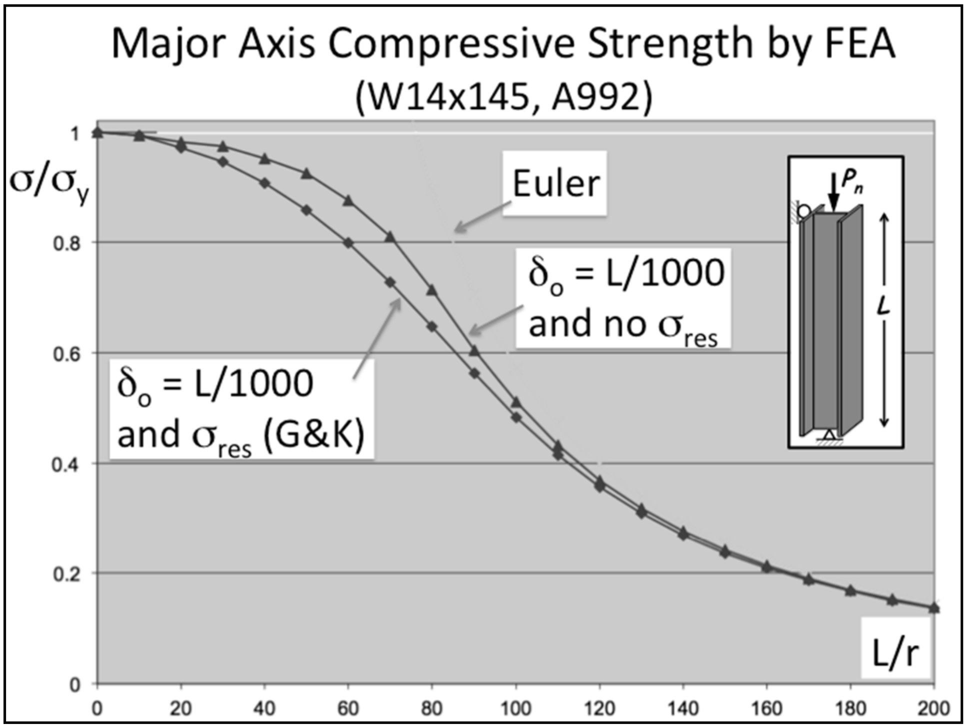


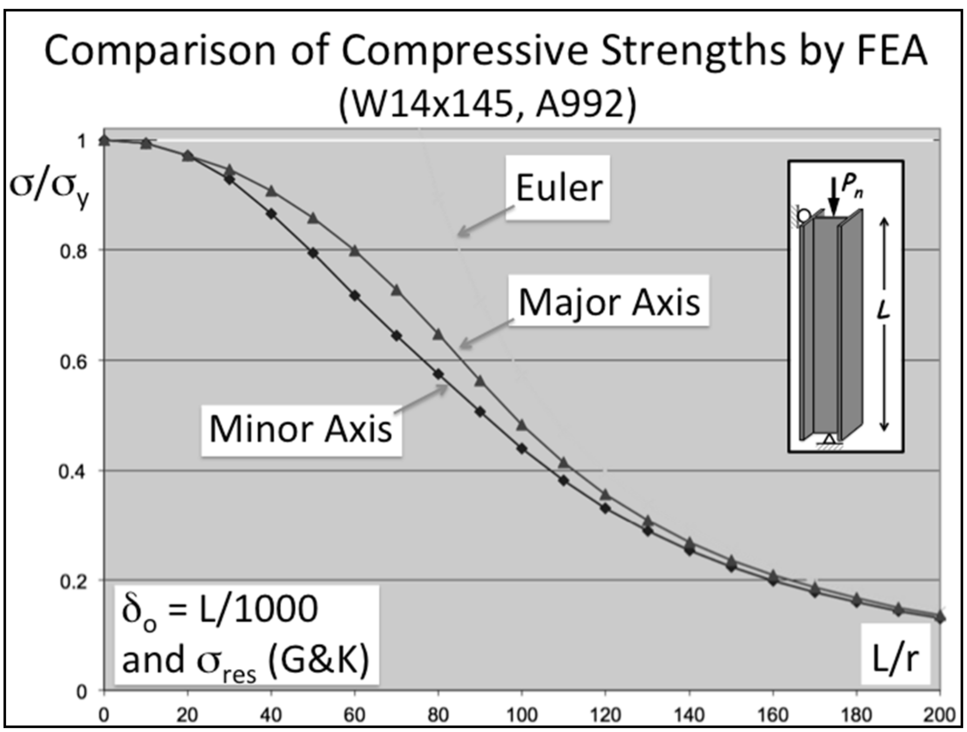
W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{cr,MNA}$	$P_{cr,GNA}$	$P_{cr,GMNIA}$
50	199			
150	597			

Minor Axis Compressive Strength by FEA
(W14x145, A992)



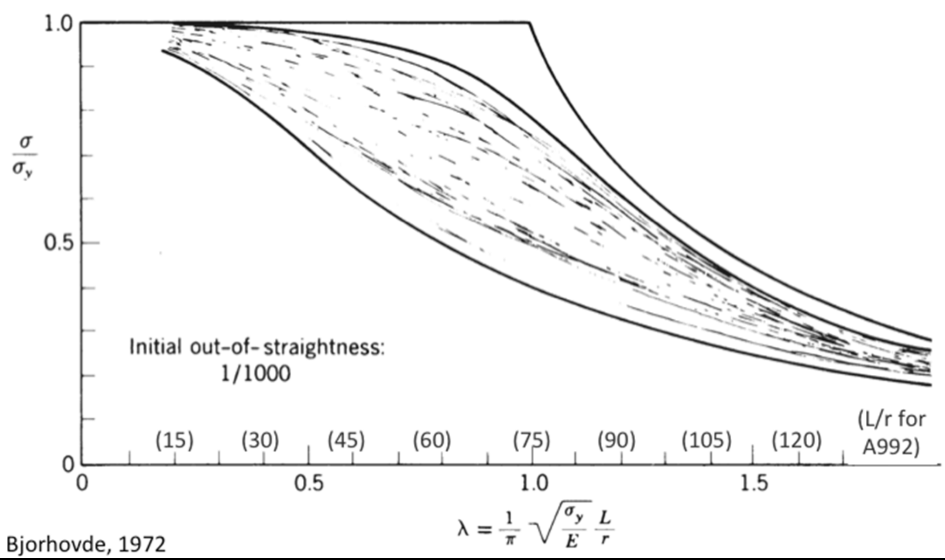

Major Axis Compressive Strength by FEA
(W14x145, A992)
σ/σy
Euler
δo = L/1000
and no σres
δo = L/1000
and σres (G&K)
Pn
L
L/r
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
100
120
140
160
180
200


Comparison of Compressive Strengths by FEA
(W14x145, A992)
σ/σy
Euler
Major Axis
Minor Axis
δo = L/1000
and σres (G&K)
Pn
L
L/r
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
100
120
140
160
180
200

Compressive Strength Curves

- Key observations from FEA
  - Strength reduced for initial imperfection and further reduced for residual stresses
  - All curves approach Euler, but are slightly below
  - Partial yielding accentuated by residual stresses impact minor axis strength more than major axis strength
  - Different strength curves for major and minor axis bending
- Additional thoughts
  - Strength curves for W-shapes are function of dimensions, and thus will vary depending on W-shape
  - Other shapes (e.g., HSS, C's, and built-up shapes) will also have different compressive strength curves

Maximum Compressive Strength Curves for Many Different Column Types

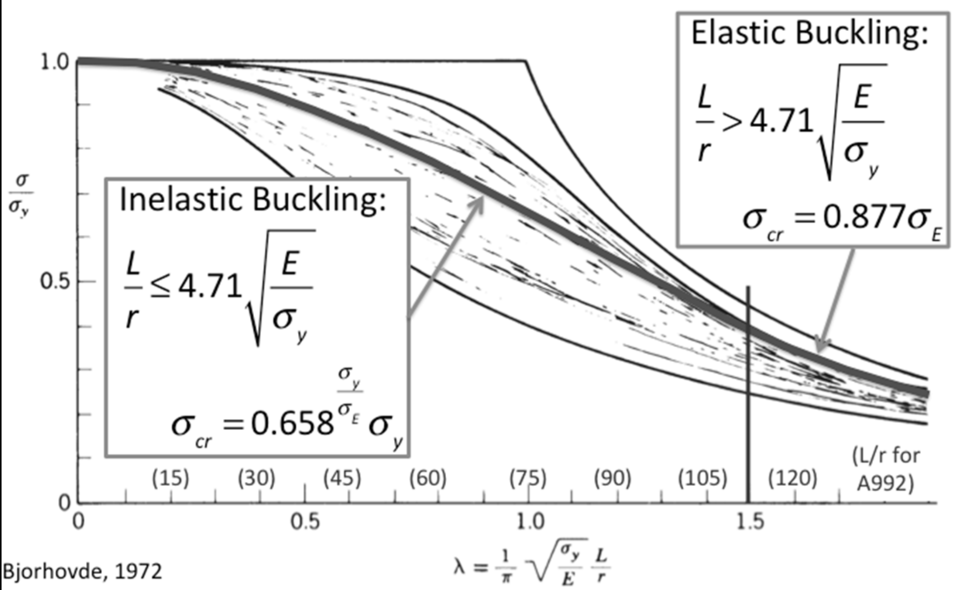


Bjorhovde, 1972

Column Curves for Design

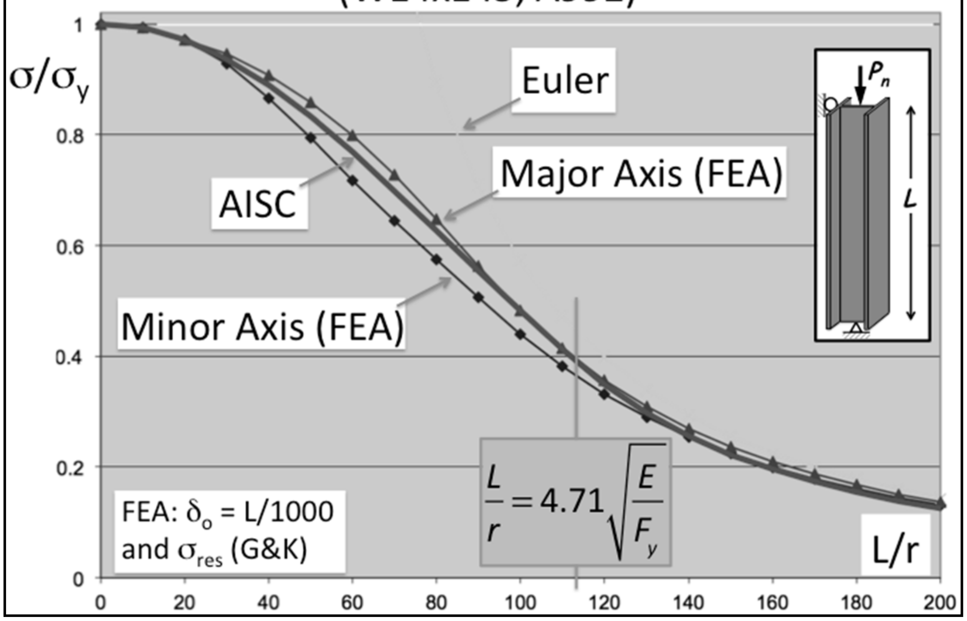
- AISC employs a single curve "fit" to experimental and analytical data. Other codes use multiple curves.
- Background to AISC curve:
  - Bjorhovde, R. (1972), "Deterministic and Probabilistic Approaches to the Strength of Steel Columns," Ph.D. Dissertation, Lehigh University, Bethlehem, PA.
  - Tide, R.H.R. (2001), "A Technical Note: Derivation of the LRFD Column Design Equations," Engineering Journal, AISC, Vol. 38, No. 3, 3rd Quarter, pp. 137–139.
  - Ziemian, R.D. (ed.) (2010), *Guide to Stability Design Criteria for Metal Structures*, 6th Ed., John Wiley & Sons, Inc., Hoboken, NJ.

AISC Column Curve:



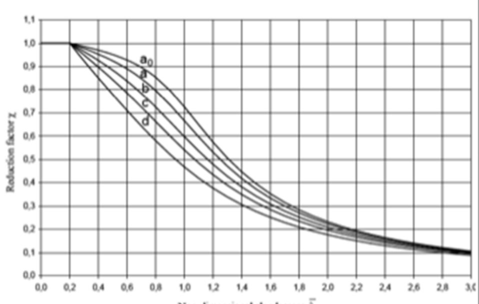
Bjorhovde, 1972

Comparison of Compressive Strengths
(W14x145, A992)



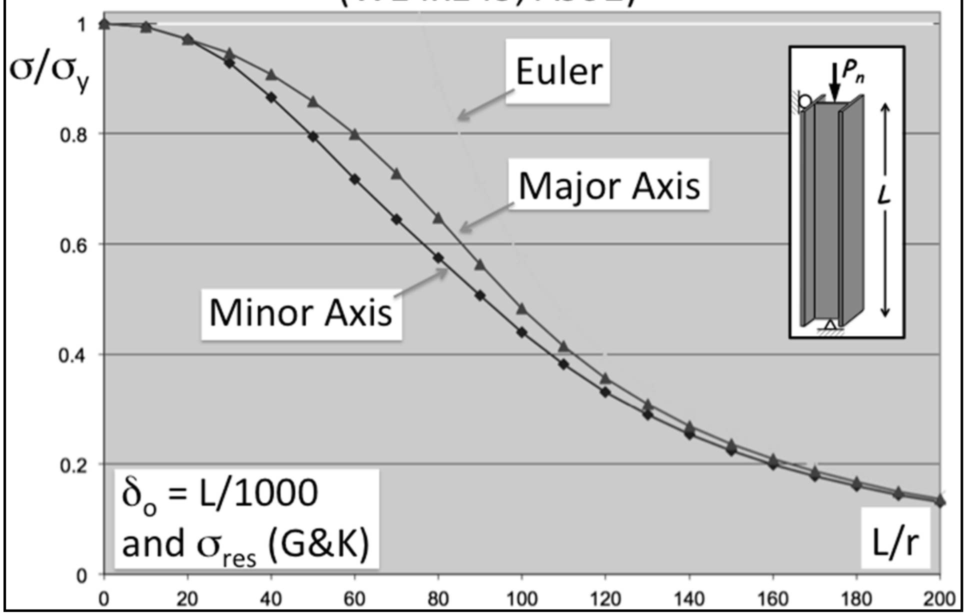
Multiple Column Curves:

Eurocode 3

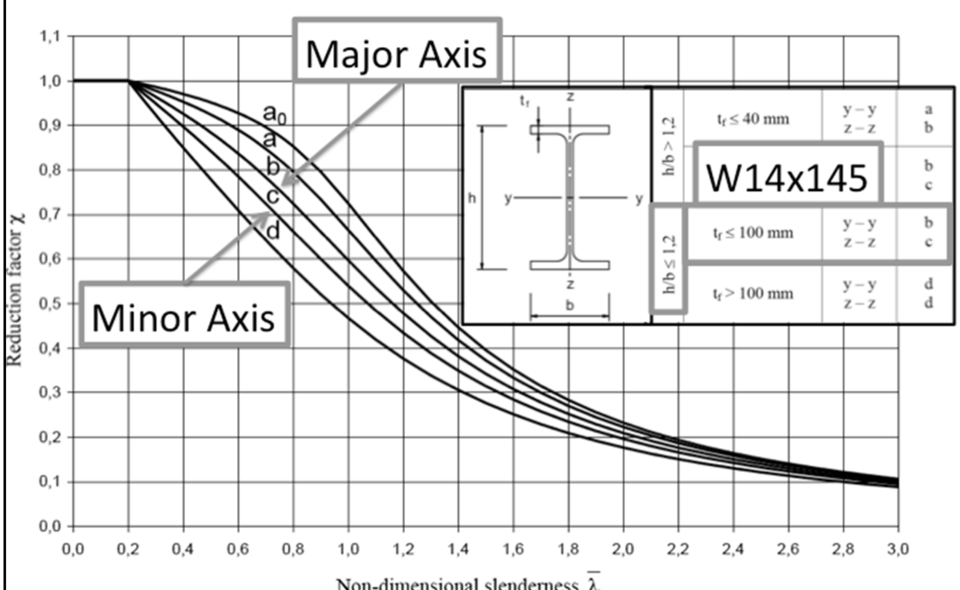


Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		$h/b > 1,2$	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			40 mm < $t_f \le 100$	y – y z – z	b c	a a
		$h/b \le 1,2$	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections		$t_f \le 40$ mm		y – y z – z	b c	b c
		$t_f > 40$ mm		y – y z – z	c d	c d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U-, T- and solid sections				any	c	c
L-sections				any	b	b

Comparison of Compressive Strengths by FEA
(W14x145, A992)

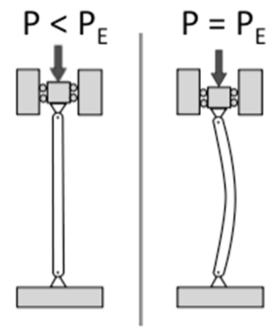


Multiple Column Curves: Eurocode 3

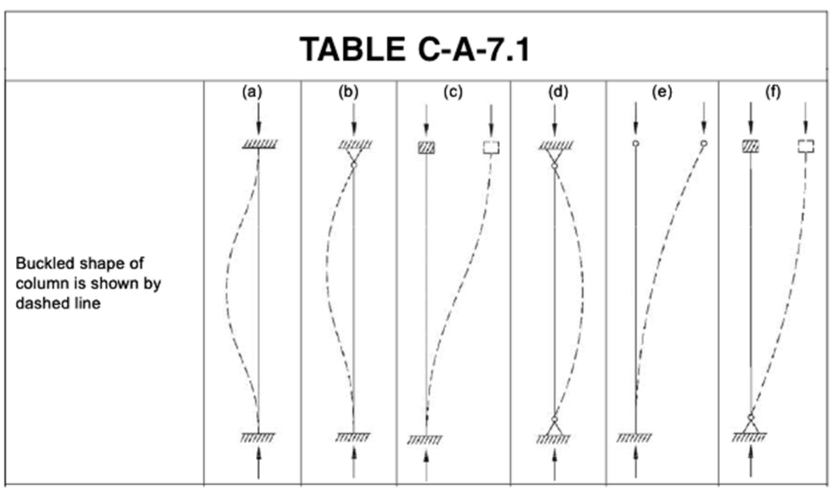


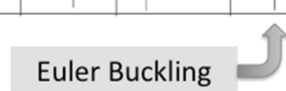
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)

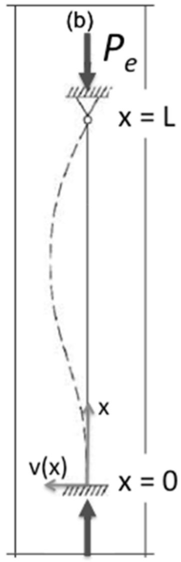


Support Conditions





What about the others?



Support Conditions (2)

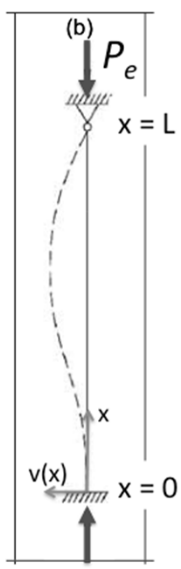
Equilibrium → Differential Equation:

$$M(x=0)=M_o \quad \Rightarrow \quad EI\frac{d^2v}{dx^2}+P_e v=\frac{M_o x}{L}$$

Solution:

$$v(x)=C_1\cos\left(\sqrt{\frac{P_e}{EI}}x\right)+C_2\sin\left(\sqrt{\frac{P_e}{EI}}x\right)+\frac{M_o x}{P_e L}$$

wolframalpha.com
a2*y"(x)+a1*y(x)=a3*x



Support Conditions (3)

Equilibrium → Differential Equation:

$$M(x=0)=M_o \quad \Rightarrow \quad EI\frac{d^2v}{dx^2}+P_e v=\frac{M_o x}{L}$$

Solution:

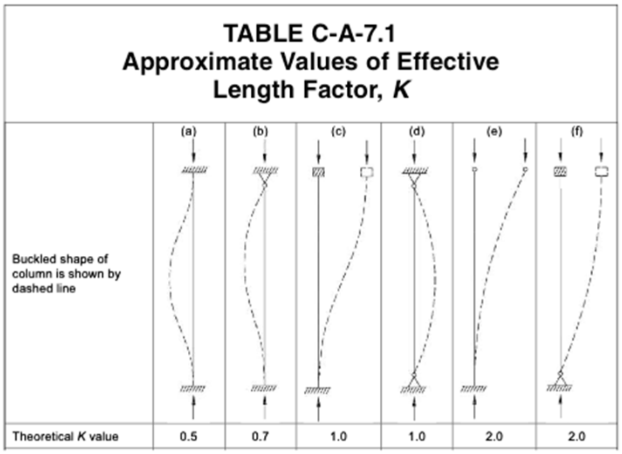
$$v(x)=C_1\cos\left(\sqrt{\frac{P_e}{EI}}x\right)+C_2\sin\left(\sqrt{\frac{P_e}{EI}}x\right)+\frac{M_o x}{P_e L}$$

Boundary Conditions:

$$v(x=0)=0,\ v'(x=0)=0,\ v(x=L)=0$$

$$P_e=\frac{\pi^2 EI}{\left(0.70L\right)^2} \quad \Rightarrow \quad \sigma_e=\frac{P_e}{A}=\frac{\pi^2 E}{\left(KL/r\right)^2} \text{ with } K=0.70$$

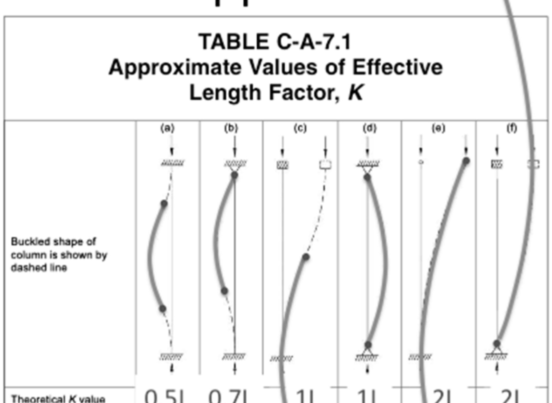
Support Conditions (4)



Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Support Conditions (5)

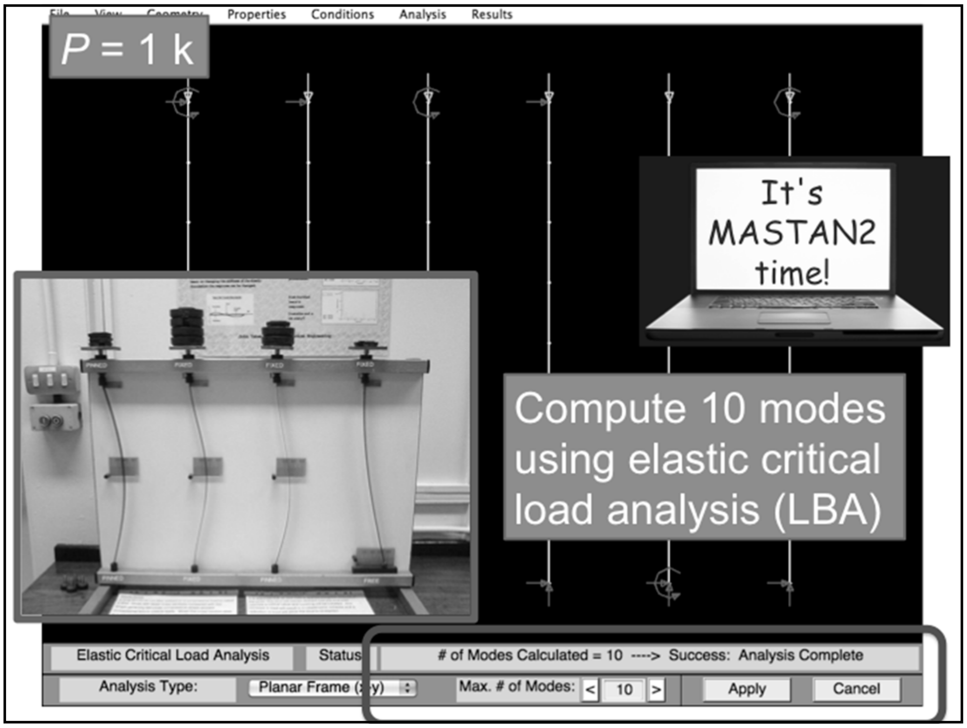


Elastic Buckling Stress:

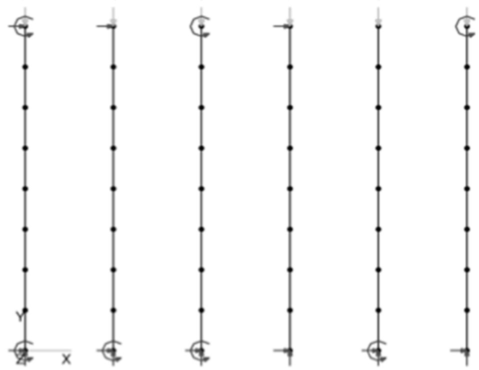
$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Notes on "effective length" *KL*:

- Find the Euler column?!
- Distance between inflection points (M=0)



Case: a b c d e f

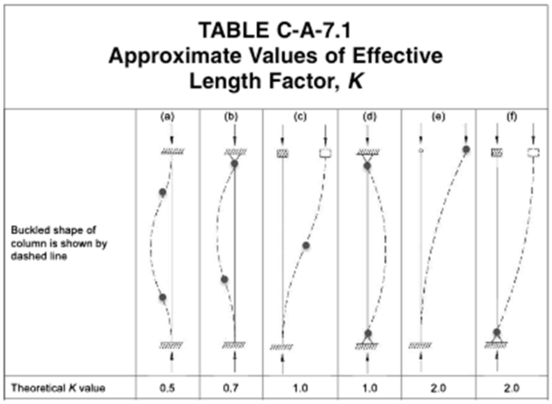


Compute $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$

and $K = \dfrac{\pi}{L}\sqrt{\dfrac{EI}{P_{cr}}}$

Case	Column End Restraints	Theoretical P_{cr}	Analysis P_{cr}	% difference	Theoretical K	Analysis K	% difference	Sort Order
a	fixed—fixed							
b	fixed—pinned							
c	fixed—free							
d	pinned—pinned							
e	fixed—free							
f	pinned—no rot.							

Support Conditions (6)



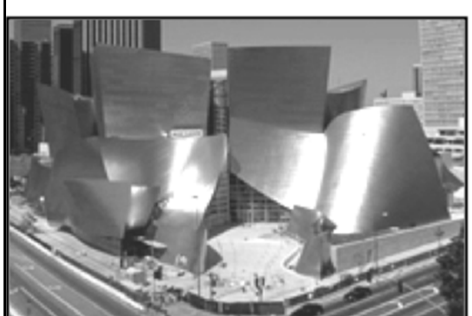
Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

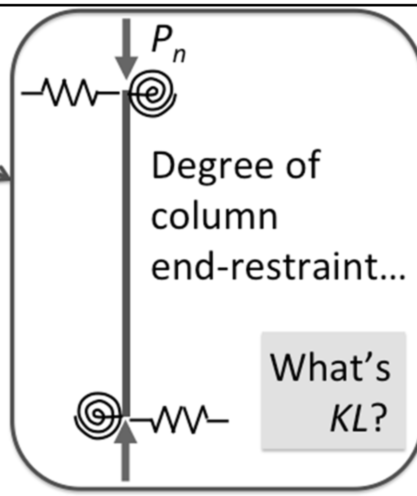
Notes on "effective length" *KL*:

- Distance between inflection points (M=0)
- Function of degree of column end-restraint
- Degree of column end-restraint can be difficult to compute accurately in real structures (hmmm…)

Support Conditions (7)



Walt Disney Concert Hall

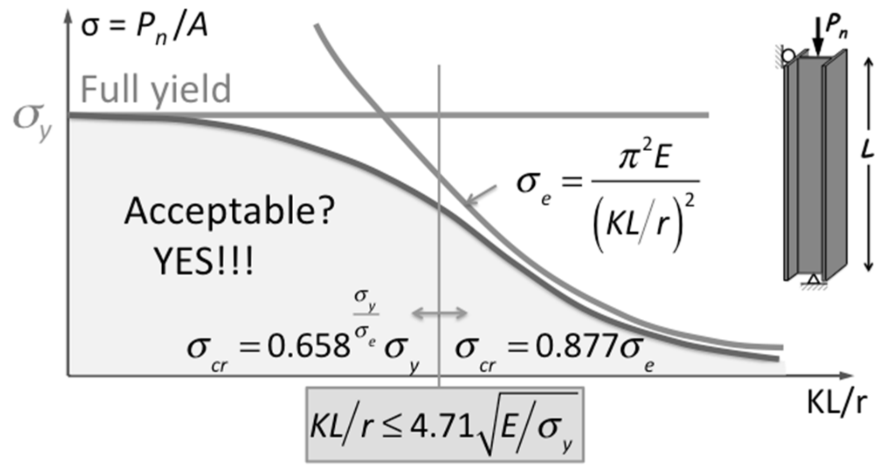


Possible solutions:

- Diff. Eq./Eigenvalue FEA
- Alignment charts (careful!)

Support Conditions (4)

- Degree of column end restraint accounted for by use of "effective length" KL (i.e., $\sigma_E \rightarrow \sigma_e$)
- AISC Column Curve – Final Take!



Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded

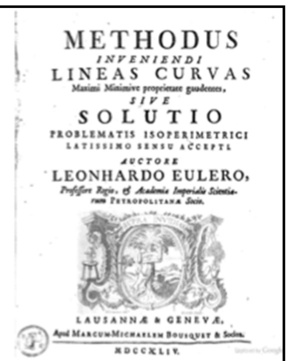
Undone

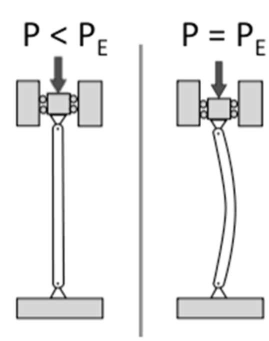
  - linear elastic behavior (E = constant)

Undone

  - pinned-roller supports (frictionless)

Undone





Summary

- Course introduction and stability concepts
- Limit states of compression members with focus on flexural buckling
- Euler Buckling → Maximum Compressive Strength Column Curve
- Column curves in codes account for:
  - full yielding
  - bending due to initial imperfection (out-of-straightness)
  - partial yielding accentuated by presence of residual stresses
  - degree of end restraint

Summary(2)

- AISC and Eurocode column curves discussed
- Other ideas introduced, including
  - moment amplification factor (2^{nd}-order effects)
  - stiffness reduction τ-factor
  - Difficulty in computing K-factors…

Behavior of Flexural Members

Ron Ziemian

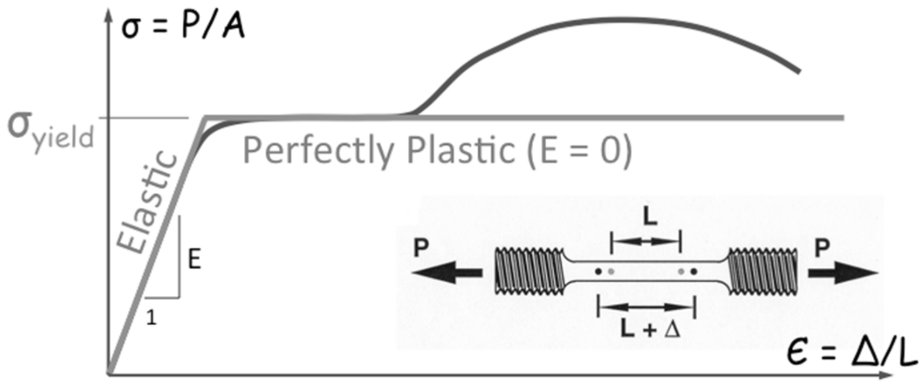
Lectures 5 & 6: 14-Aug-2014

Limit States of Flexural Members

- Full yielding (today)
- Instability
  - Along the member length (today's emphasis!)
    - Lateral torsional buckling
      - elastic
      - inelastic
  - At the cross section
    - local buckling

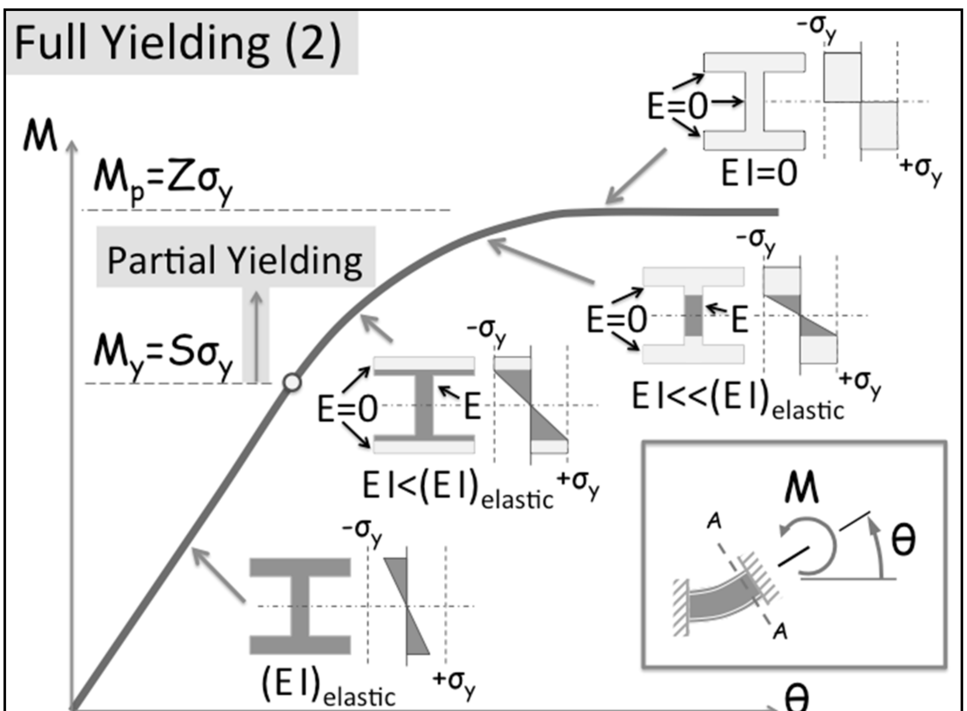
Full Yielding

- Tensile test

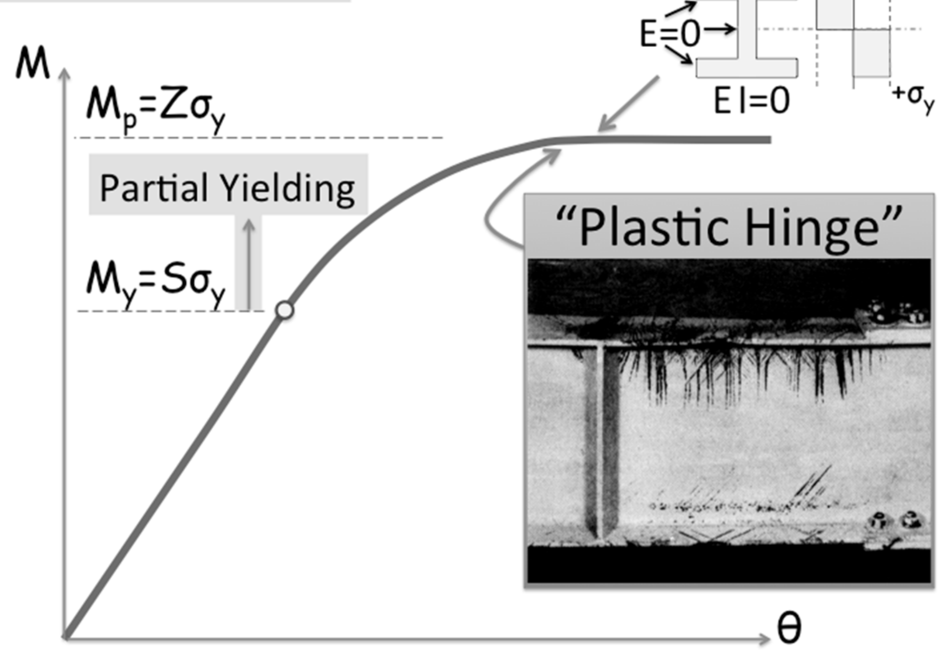


- Assume same response for compression
  - $\sigma_{y,\text{compression}} = \sigma_{y,\text{tension}} = \sigma_{\text{yield}}$
  - Neglect strain hardening (assume elastic-plastic)

Full Yielding (2)

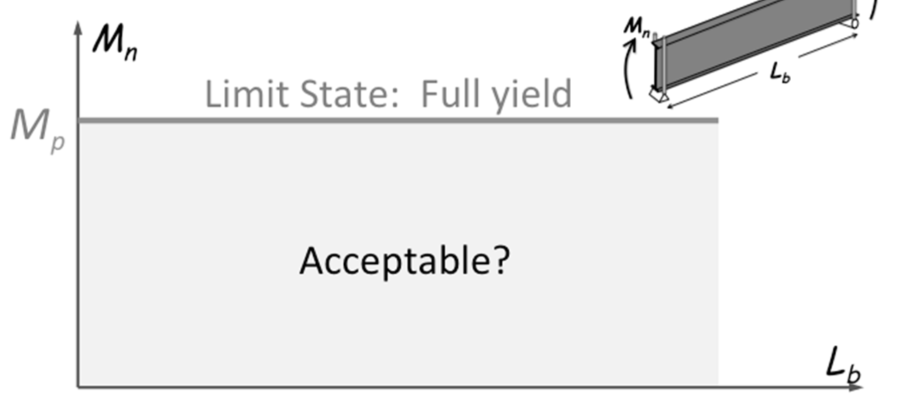


Full Yielding (3)

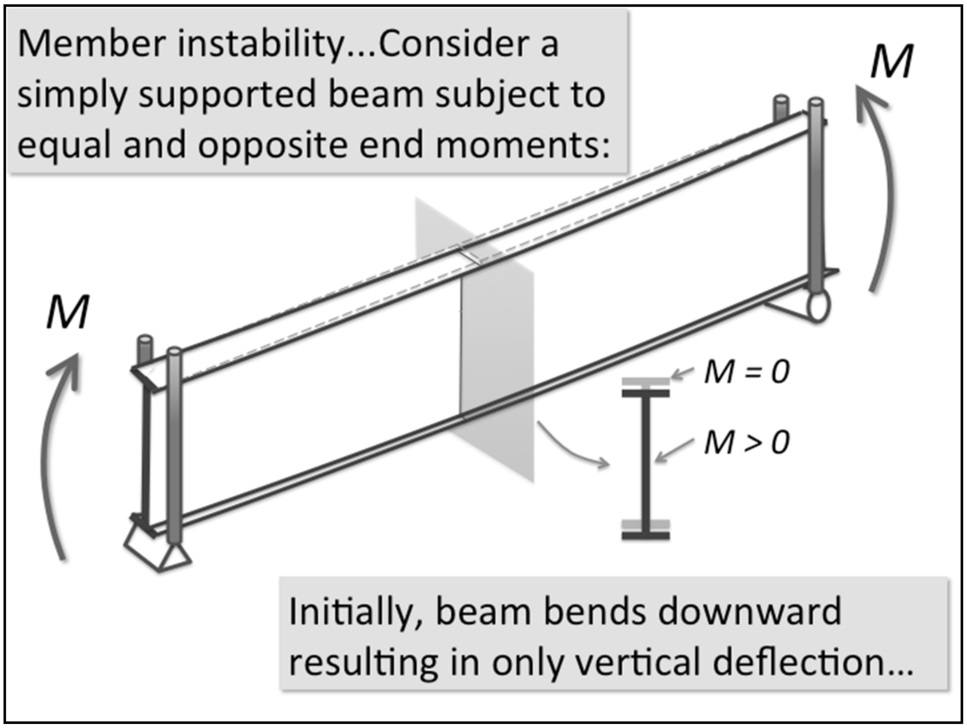


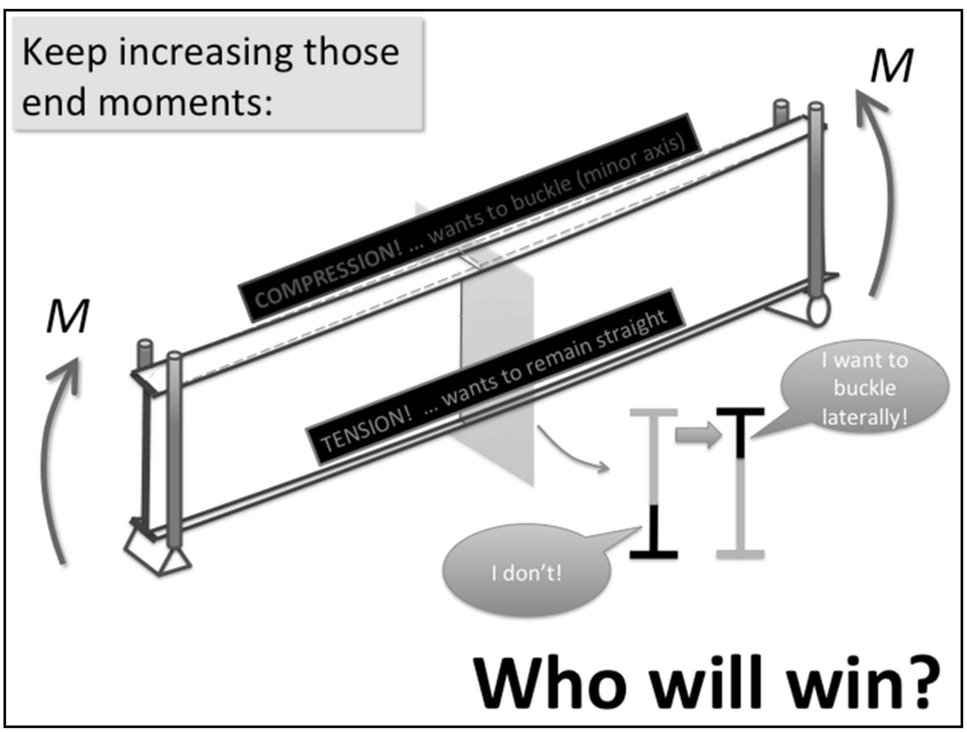
Full Yielding (4)

- Beam Curve – Take 1

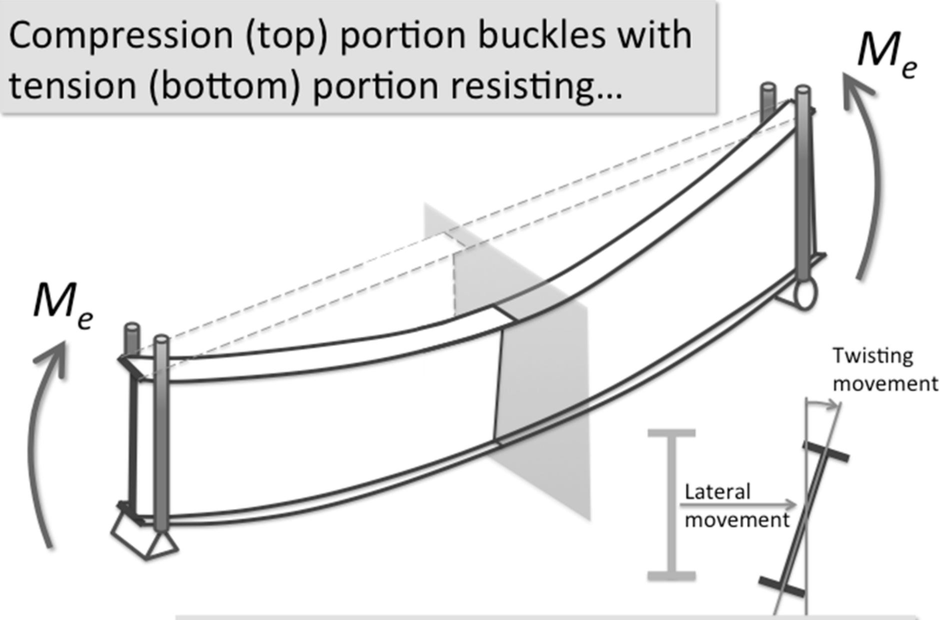


- What about:
  - member instability ??? (today!)
  - cross section instability (local buckling) ???


Member instability...Consider a simply supported beam subject to equal and opposite end moments:
M
M
M = 0
M > 0
Initially, beam bends downward resulting in only vertical deflection...

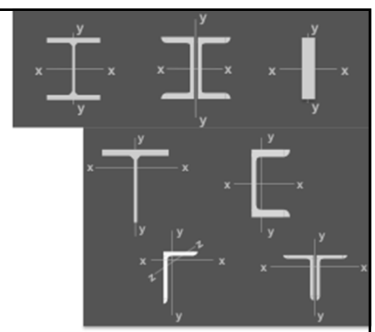

Keep increasing those end moments:
M
M
COMPRESSION! ... wants to buckle (minor axis)
TENSION! ... wants to remain straight
I want to buckle laterally!
I don't!
Who will win?

Compression (top) portion buckles with tension (bottom) portion resisting…

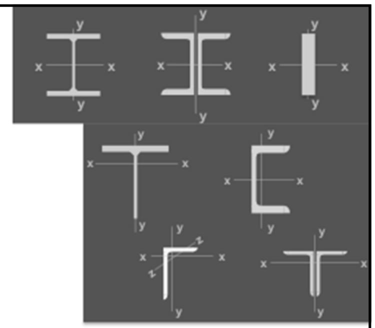


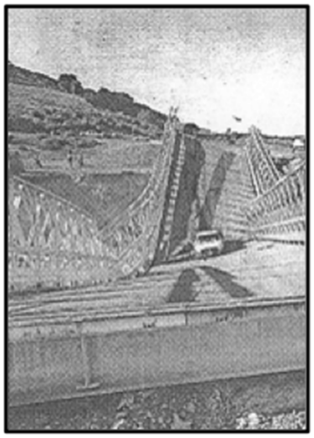
"Lateral torsional buckling (LTB)"

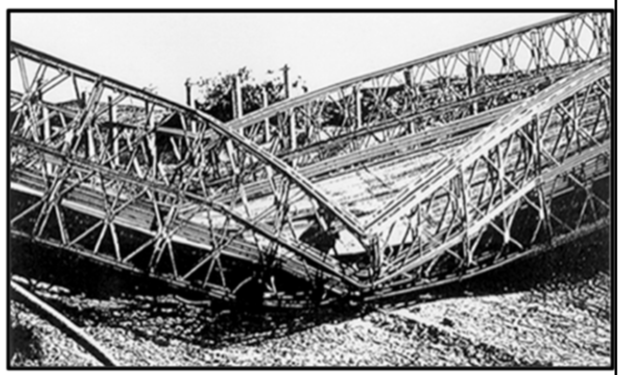
Member instability:
Lateral Torsional Buckling



Member instability: Lateral Torsional Buckling





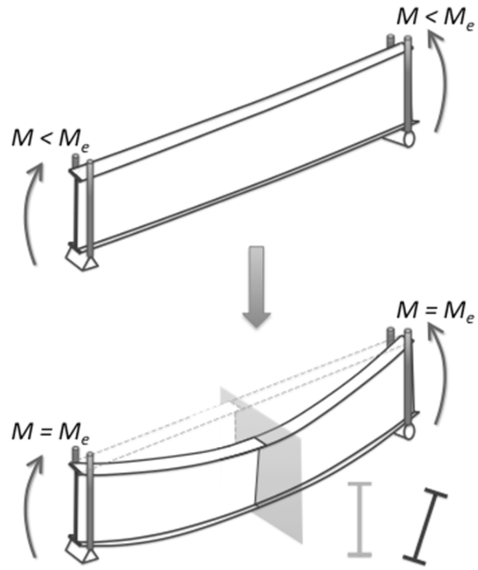


Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC and Eurocode
  - others

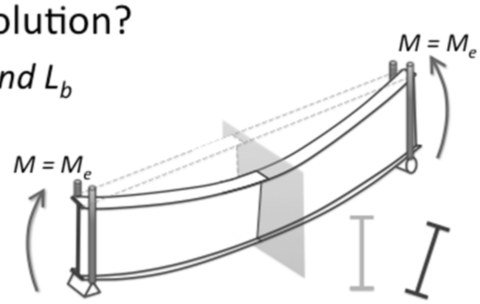
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



LTB (2)

- Before obtaining the theoretical solution for M_e, let's do a "parametric" analysis…
- Terms expected in the solution?
  - Minor axis buckling: EI_y *and* L_b
  - Torsion
    - St. Venant: GJ and L_b
    - Warping: EC_w and L_b
  - Others? π (of course!)



- What's their impact?

Terms in numerator

Term in denominator

Material: $E\uparrow, G\uparrow \;\Rightarrow\; M_e\uparrow$

Section: $I_y\uparrow, J\uparrow, C_w\uparrow \;\Rightarrow\; M_e\uparrow$

Unbraced length: $L_b\uparrow \;\Rightarrow\; M_e\downarrow$

Wait…

- Minor axis buckling, I recall from Sessions 1 and 2

$$P_E = \frac{\pi^2 EI}{L^2}$$

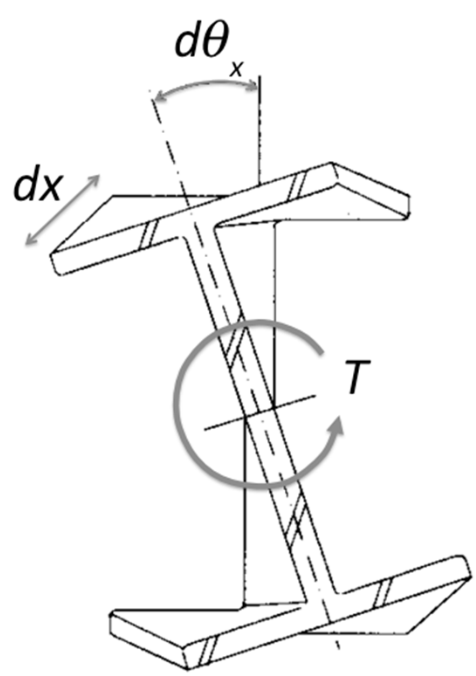
- But, I need a quick refresher on torsion!

St. Venant ?
Warping ????

St. Venant Torsion

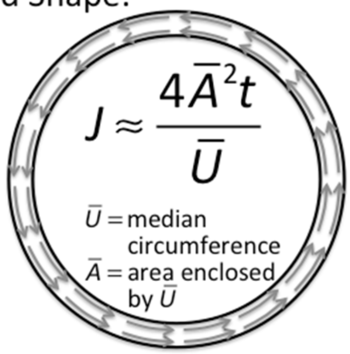
Consider a portion of the member of length *dx* subject to a torque *T*.
If we consider only St. Venant (uniform) torsion, the rotation per unit length is:

$$\frac{d\theta_x}{dx} = \frac{T}{GJ}$$



St. Venant Torsion (2)

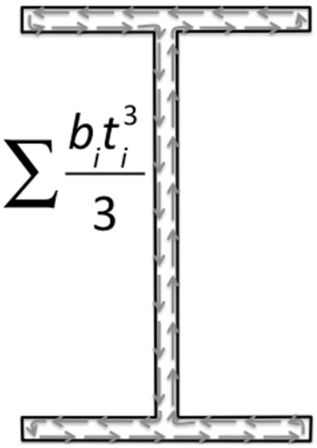
Closed Shape:



$$J \approx \frac{4\bar{A}^2 t}{\bar{U}}$$

$\bar{U}$ = median circumference
$\bar{A}$ = area enclosed by $\bar{U}$

Open Shape:



$$J \approx \sum \frac{b_i t_i^3}{3}$$

Circular Hollow Shape:
t = 0.25", A = 3.84 in^2
$D = A/(\pi t)$ = 4.90"

$$J = \frac{4(\pi \bar{D}^2/4)^2 t}{\pi \bar{D}} = 22.95 \text{ in}^4$$

W8x13 (t_f=0.23", t_w=0.26"):
A = 3.84 in^2
J = 0.0871 in^4

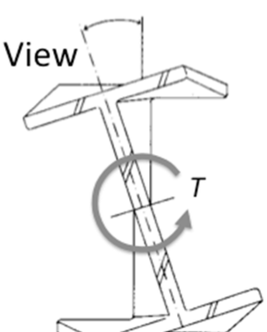
Factor of 264…closed sections rule in torsion!

Warping Torsion (your new best friend!)

Top View

Notice that this torque T also causes the flanges to bend in opposite directions. This "cross flange" bending can also resist the applied torque.

End View



Warping Torsion (2)

Relationship to rotation?

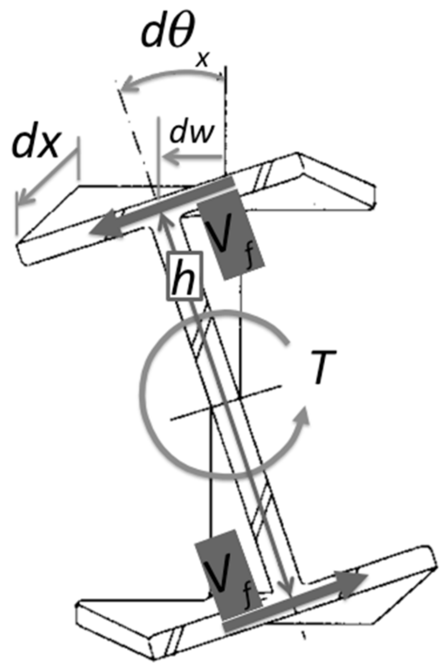
$$T = 2V_f \frac{h}{2} = V_f h$$

$$\text{with } V_f = \frac{dM_f}{dx}$$

$$M_f = -EI_f \frac{d^2 w}{dx^2}$$

$$dw = \frac{h}{2}\, d\theta_x$$

$$\boxed{T = \left(-EI_f \frac{h}{2} \frac{d^3\theta_x}{dx^3} \right) h}$$



Warping Torsion (3)

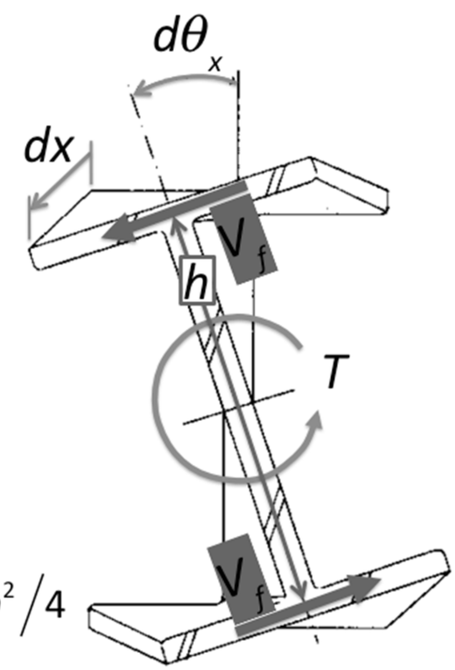
Relationship to rotation?

$$T = V_f h$$

$$T = \left(-EI_f \frac{h}{2} \frac{d^3\theta_x}{dx^3} \right) h$$

$$\boxed{T = -EC_w \frac{d^3\theta_x}{dx^3}}$$

with $I_f = I_y / 2,\ C_w = I_y h^2 / 4$



The twist on torsion

St. Venant (uniform):

$$T_{SV} = GJ\frac{d\theta_x}{dx}$$

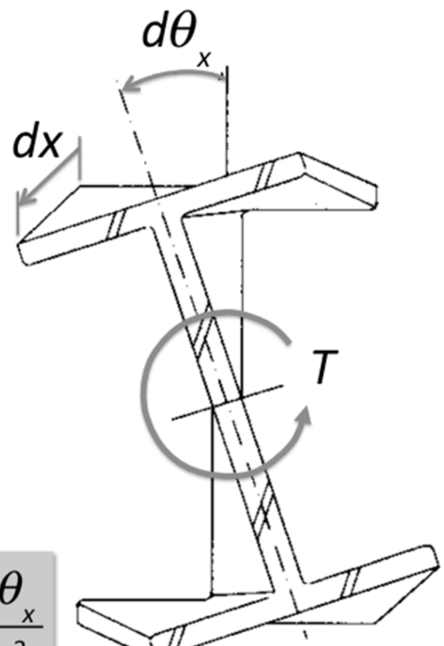
Warping(non-uniform):

$$T_w = -EC_w\frac{d^3\theta_x}{dx^3}$$

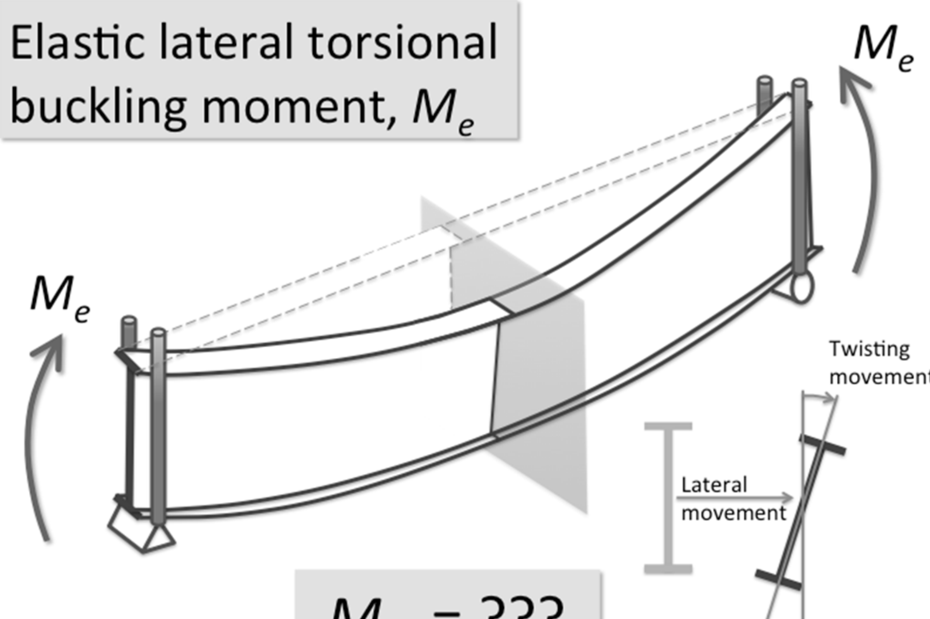
Total resisting torque:

$$T = T_{SV} + T_w$$

$$T = GJ\frac{d\theta_x}{dx} - EC_w\frac{d^3\theta_x}{dx^3}$$

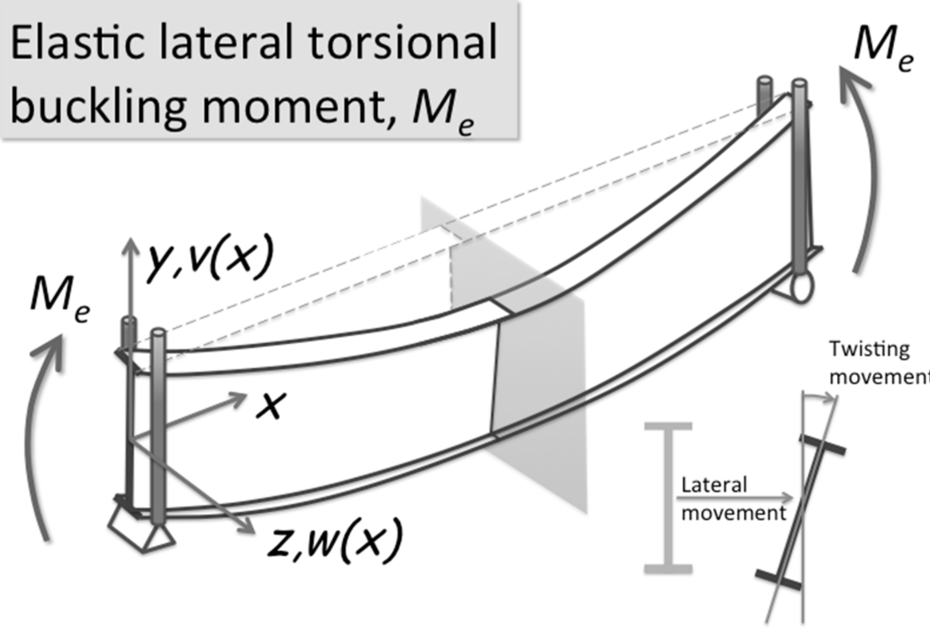


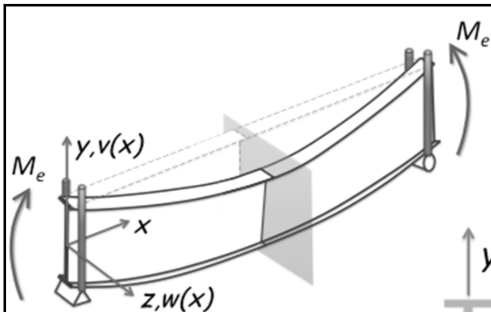
Elastic lateral torsional buckling moment, M_e



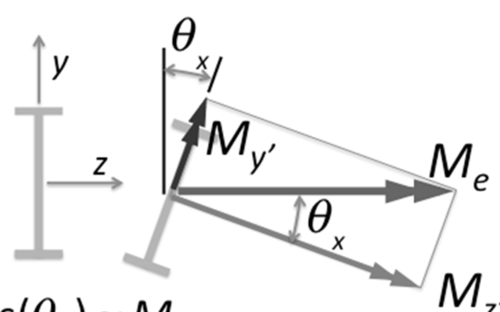
M_e = ???

Elastic lateral torsional buckling moment, M_e





Section View

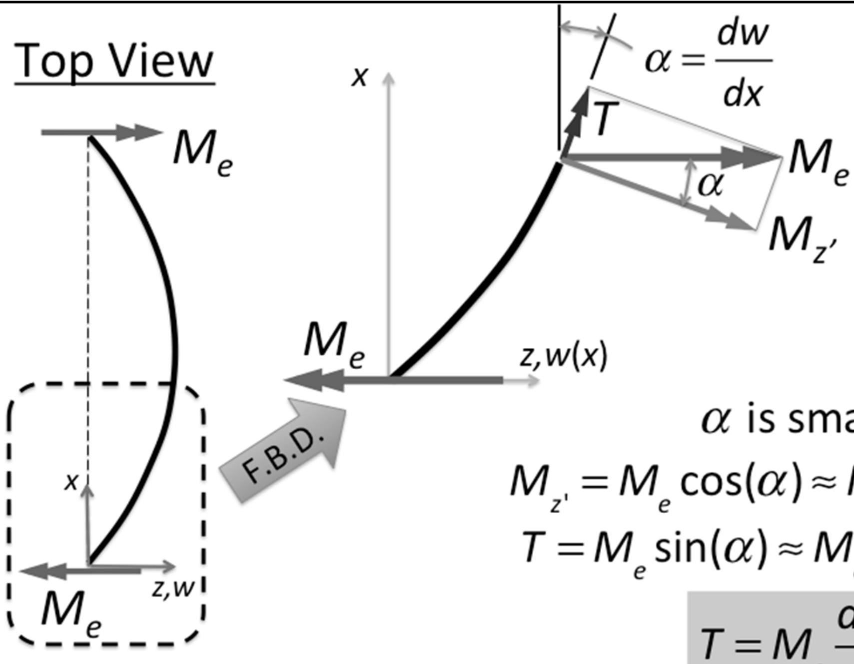


θ_x is small!

$$M_{z'} = M_e \cos(\theta_x) \approx M_e$$

$$M_{y'} = M_e \sin(\theta_x) \approx M_e \theta_x$$

$$M_{y'} = M_e \theta_x$$



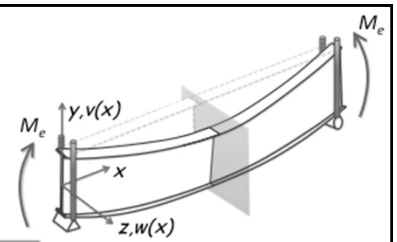
α is small!

$$M_{z'} = M_e \cos(\alpha) \approx M_e$$

$$T = M_e \sin(\alpha) \approx M_e \alpha$$

$$\boxed{T = M_e \frac{dw}{dx}}$$

Summary



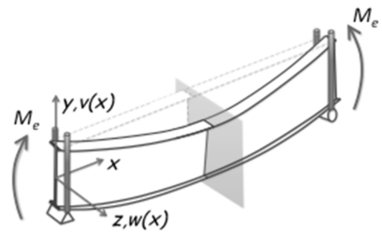
Equilibrium:

$$\underbrace{T = M_e \frac{dw}{dx}}_{\text{“applied” torque}} = \underbrace{GJ \frac{d\theta_x}{dx} - EC_w \frac{d^3\theta_x}{dx^3}}_{\text{“resisting” torque}}$$

$$\frac{d}{dx}\left(M_e \frac{dw}{dx} \right) = \frac{d}{dx}\left(GJ \frac{d\theta_x}{dx} - EC_w \frac{d^3\theta_x}{dx^3} \right)$$

$$\boxed{M_e \frac{d^2w}{dx^2} = GJ \frac{d^2\theta_x}{dx^2} - EC_w \frac{d^4\theta_x}{dx^4}}$$

Summary (2)



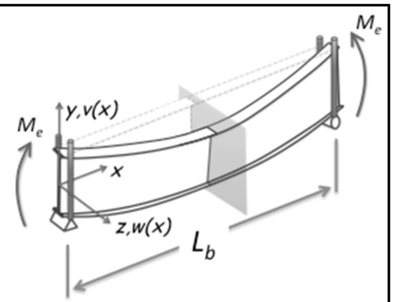
$$M_e \frac{d^2 w}{dx^2} = GJ \frac{d^2 \theta_x}{dx^2} - EC_w \frac{d^4 \theta_x}{dx^4}$$

$$M_{y'} = M_e \theta_x = -EI_{y'} \frac{d^2 w}{dx^2} \quad \Rightarrow \quad \frac{d^2 w}{dx^2} = -\frac{M_e}{EI_{y'}} \theta_x$$

$$-\frac{M_e^2}{EI_{y'}} \theta_x = GJ \frac{d^2 \theta_x}{dx^2} - EC_w \frac{d^4 \theta_x}{dx^4}$$

Summary (3)

Solve differential equation



$$EC_w \frac{d^4 \theta_x}{dx^4} - GJ \frac{d^2 \theta_x}{dx^2} - \frac{M_e^2}{EI_{y'}} \theta_x = 0$$

and apply boundary conditions

$$\theta_x (x=0) = 0, \;\; \theta_x (x = L_b) = 0$$

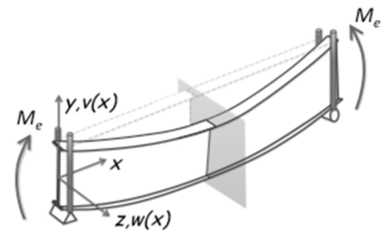
$$\theta_x^{''} (x=0) = 0, \;\; \theta_x^{''} (x = L_b) = 0$$

Results in

$$M_e^2 = \left(\frac{\pi^2 EI_y}{L_b^2} \right) \left(GJ + \frac{\pi^2}{L_b^2} EC_w \right)$$

Summary (4)

This sort of makes sense!



$$M_e^2 = \underbrace{\left(\frac{\pi^2 EI_y}{L_b^2}\right)}_{\substack{\text{Top flange in} \\ \text{compression trying} \\ \text{to produce minor} \\ \text{axis buckling}}} \underbrace{\left(GJ + \frac{\pi^2}{L_b^2} EC_w\right)}_{\substack{\text{Bottom flange in tension resisting this minor} \\ \text{axis buckling by creating a resisting torque,} \\ \text{which includes both St. Venant and Warping} \\ \text{components}}}$$

which simplifies to:

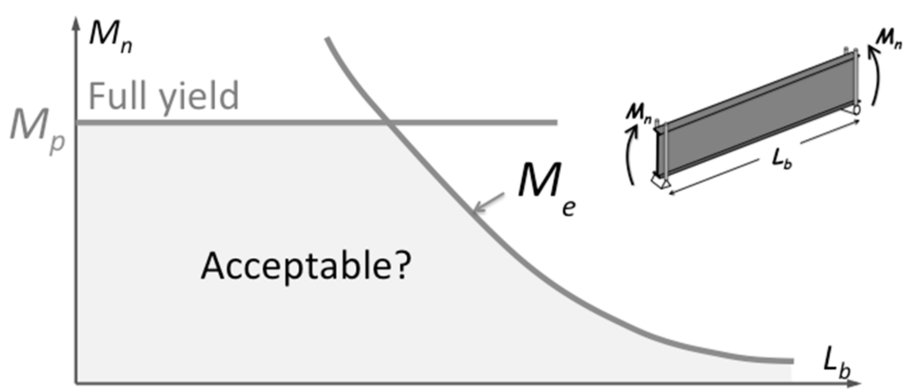
$$\boxed{M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}}$$

Also note that our earlier parametric study was spot on!

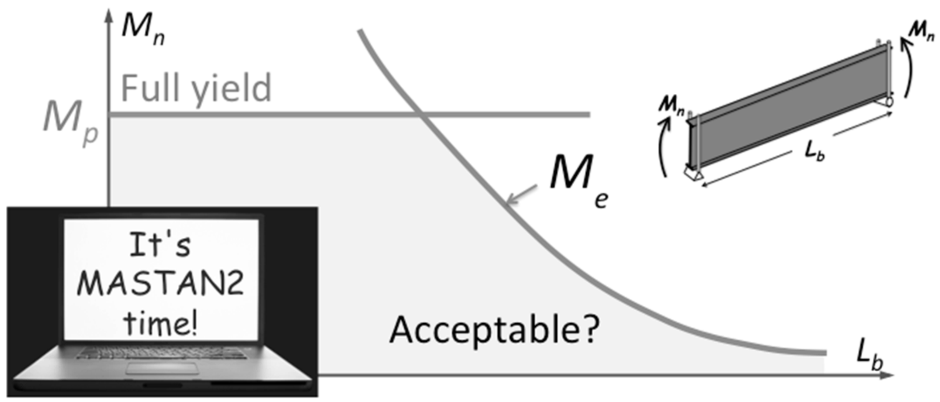
- Elastic lateral-torsional buckling

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

- Beam Curve – Take 2



- What about those assumptions?



W24x68 (E=29,000, F_y = 50, Z=177, include warping)

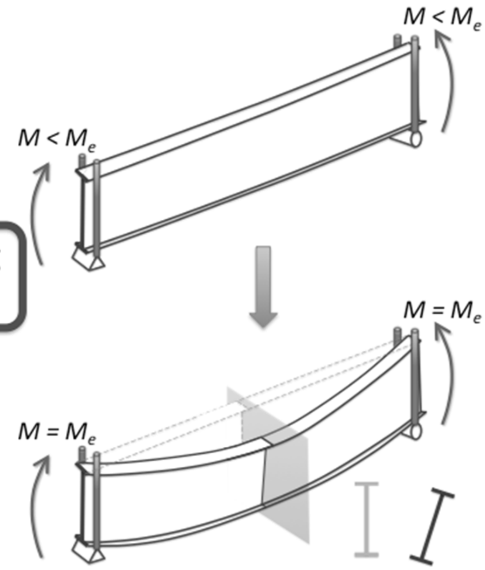
L_b	$M_{n,MNA}$	$M_{n,GNA}$	$M_{n,LBA}$	$M_{n,min}$
100				
300				

Lateral Torsion Buckling

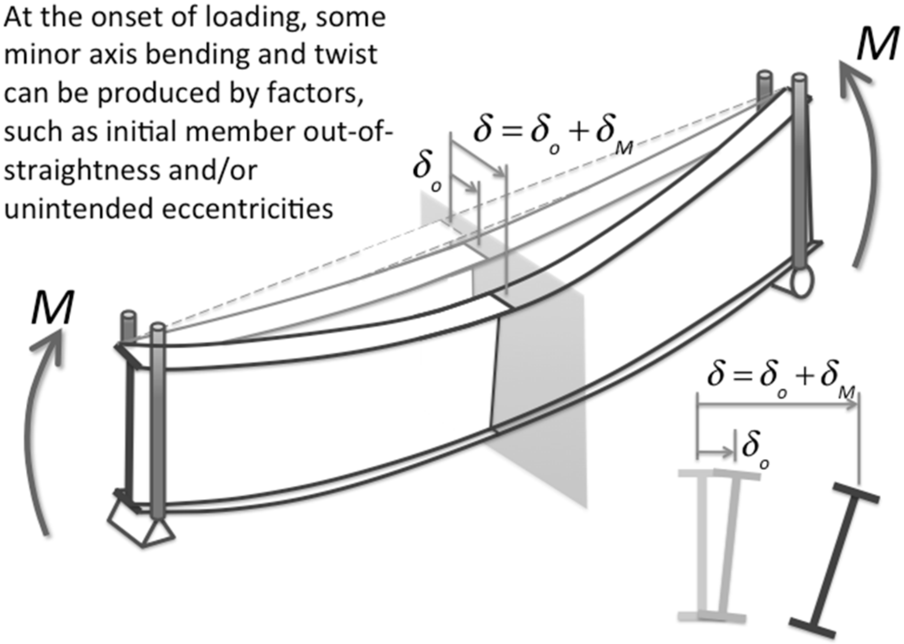
- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

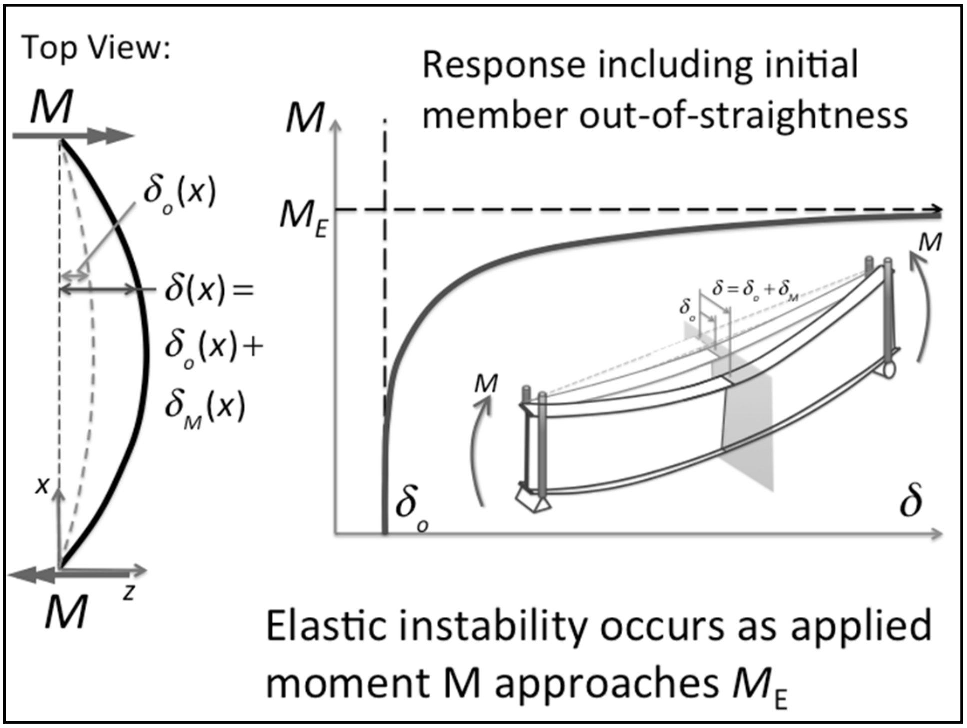
Lateral Torsional Buckling (LTB)

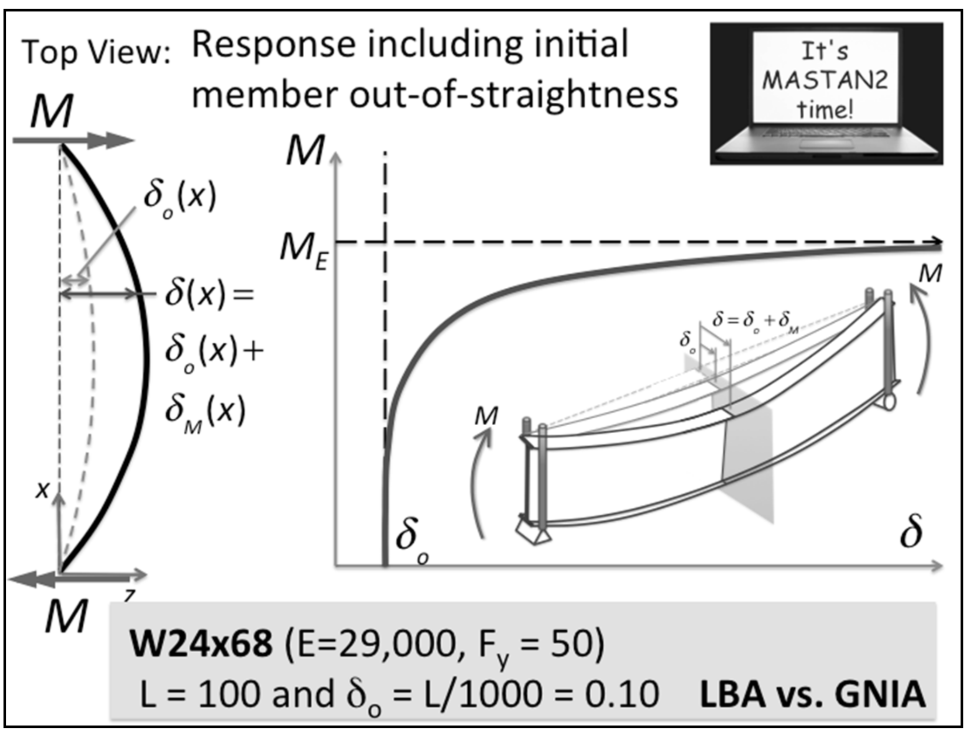
- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



At the onset of loading, some minor axis bending and twist can be produced by factors, such as initial member out-of-straightness and/or unintended eccentricities

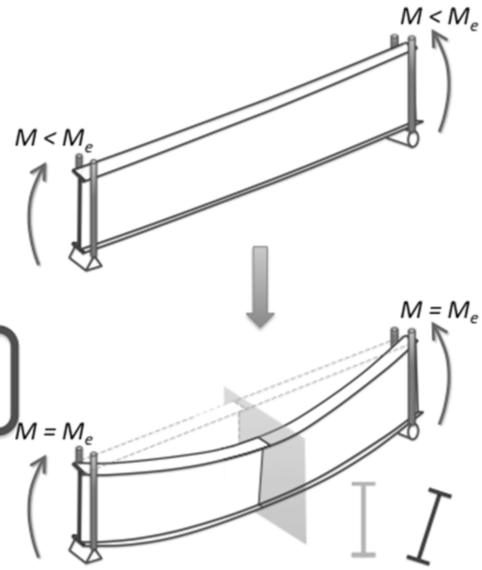



Top View:
Response including initial member out-of-straightness
M
$\delta_o(x)$
$\delta(x) = \delta_o(x) + \delta_M(x)$
x
z
M
M_E
$\delta = \delta_o + \delta_M$
δ_o
δ
Elastic instability occurs as applied moment M approaches M_E


Top View:
Response including initial member out-of-straightness
It's MASTAN2 time!
M
$\delta_o(x)$
$\delta(x) = \delta_o(x) + \delta_M(x)$
x
z
M
M_E
$\delta = \delta_o + \delta_M$
δ_o
δ
W24x68 (E=29,000, F_y = 50)
L = 100 and δ_o = L/1000 = 0.10 **LBA vs. GNIA**

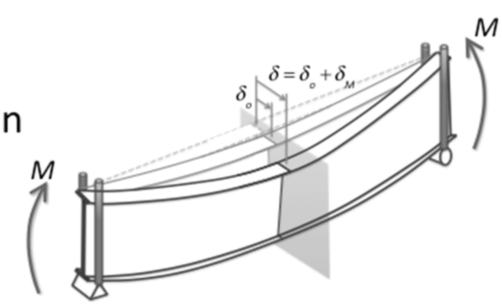
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)

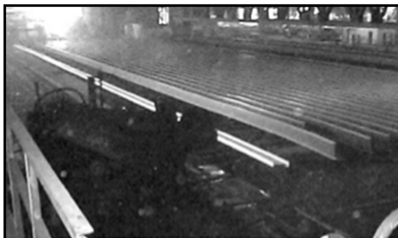


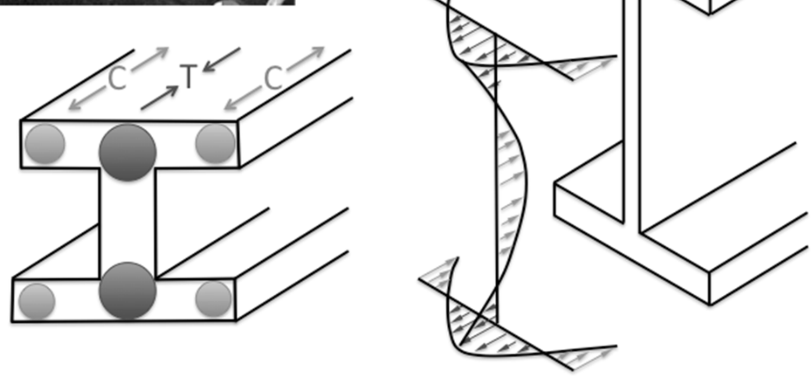
Partial Yielding

- As loading is applied, cross section may begin to yield due to
  - major axis bending
  - minor axis bending
  - torsion (warping stresses)
- Yielding is accentuated by presence of residual stresses
- Yielding results in loss of stiffness, which may result inelastic lateral torsional buckling.



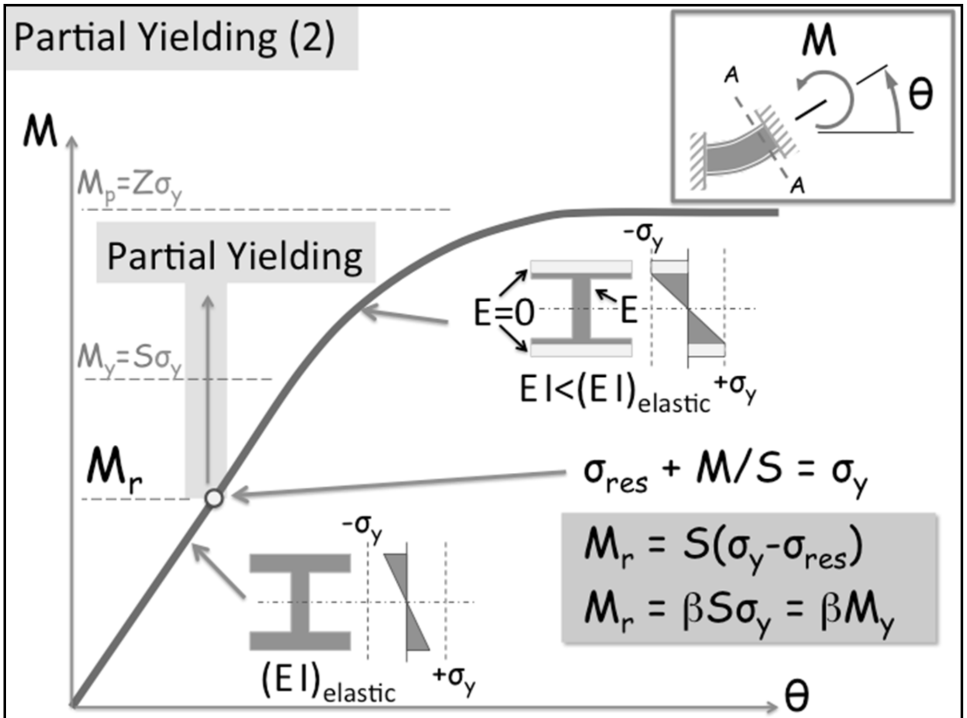
Reminder: Residual Stresses

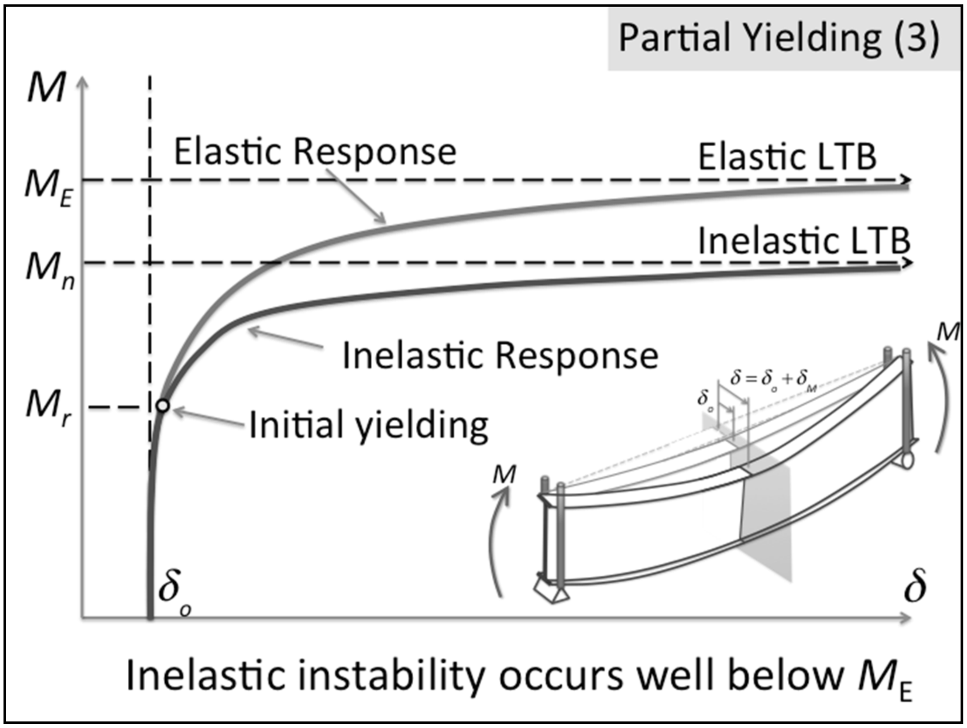


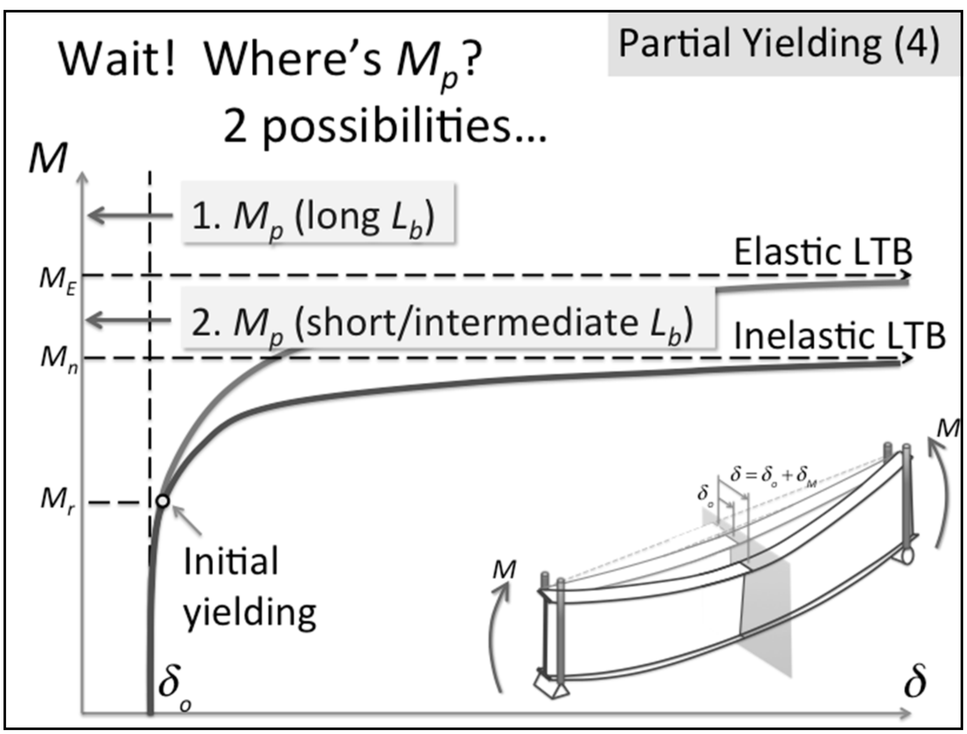


Occur in rolled wide flange shapes because locations with high surface area (e.g., flange tips) cool well before locations with smaller surface area (flange-to-web intersections)

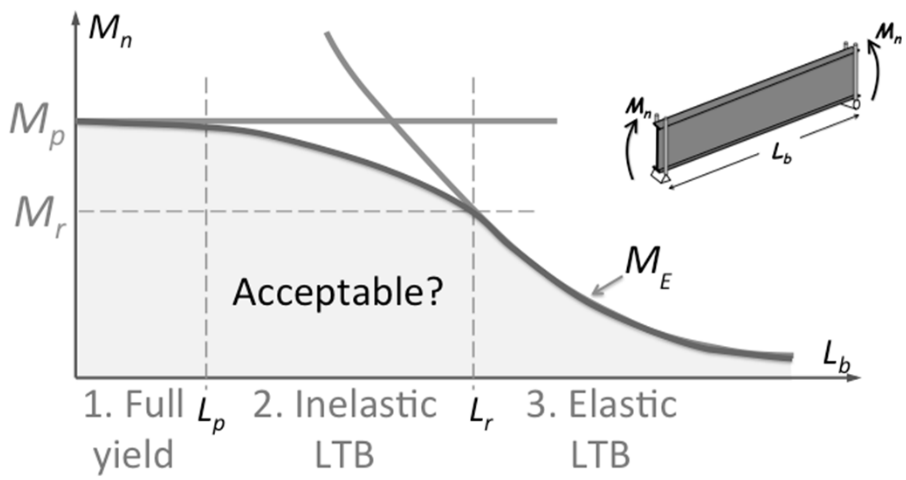
Partial Yielding (2)



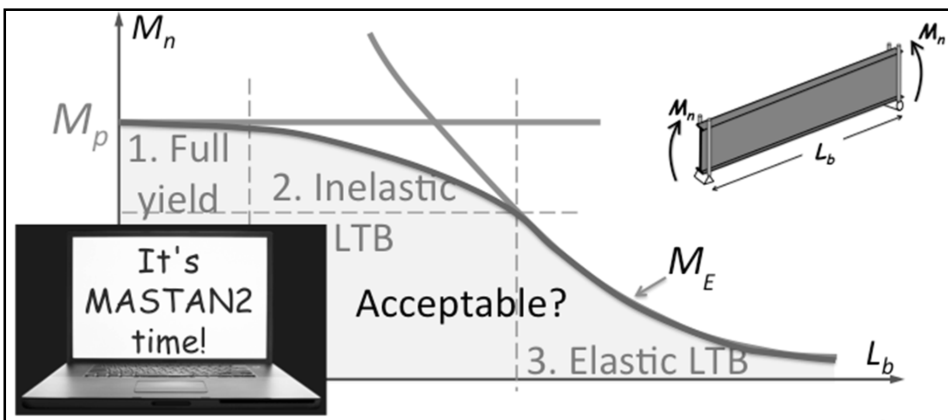

Partial Yielding (3)
M
Elastic Response
Elastic LTB
M_E
Inelastic LTB
M_n
Inelastic Response
M
$\delta = \delta_o + \delta_M$
δ_o
M_r
Initial yielding
M
δ_o
δ
Inelastic instability occurs well below M_E


Partial Yielding (4)
Wait! Where's M_p?
2 possibilities…
M
1. M_p (long L_b)
Elastic LTB
M_E
2. M_p (short/intermediate L_b)
Inelastic LTB
M_n
M
$\delta = \delta_o + \delta_M$
δ_o
M_r
Initial yielding
M
δ_o
δ

- Beam Curve – Take 3

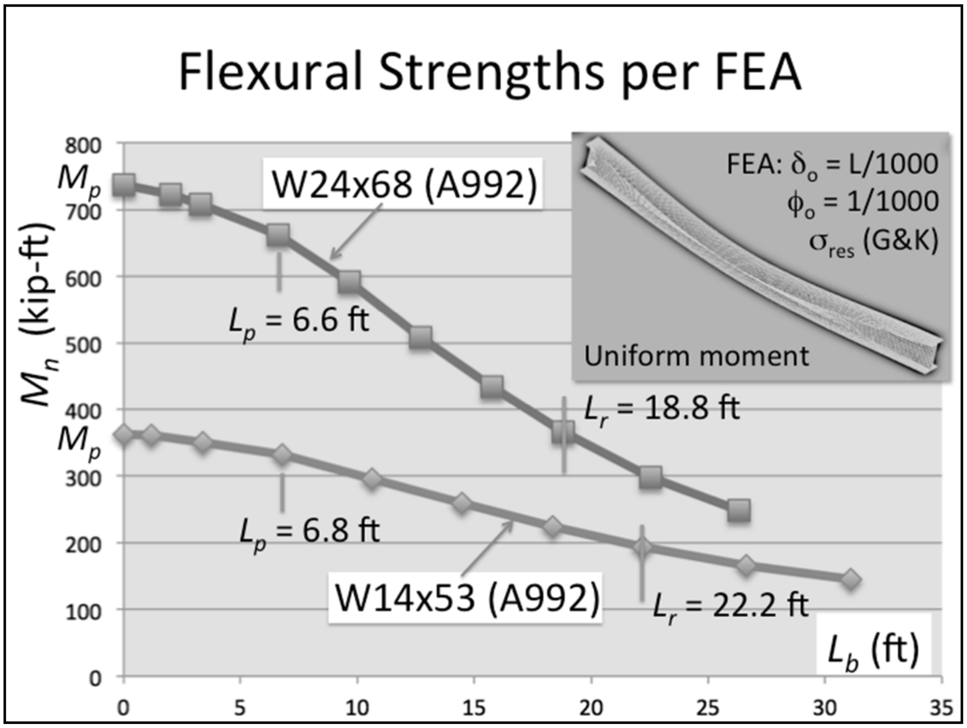


- Where did L_p and L_r come from?
- What about those assumptions?

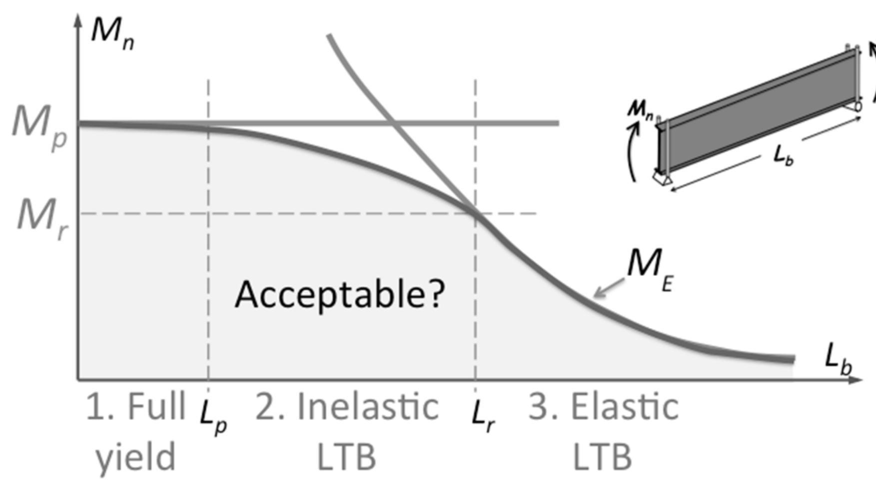


W24x68 (E=29,000, F_y = 50, include warping and δ_o)

L_b	$M_{n,MNA}$	$M_{n,GNA}$	$M_{n,GMNIA}$	$M_{n,min}$
100				
300				



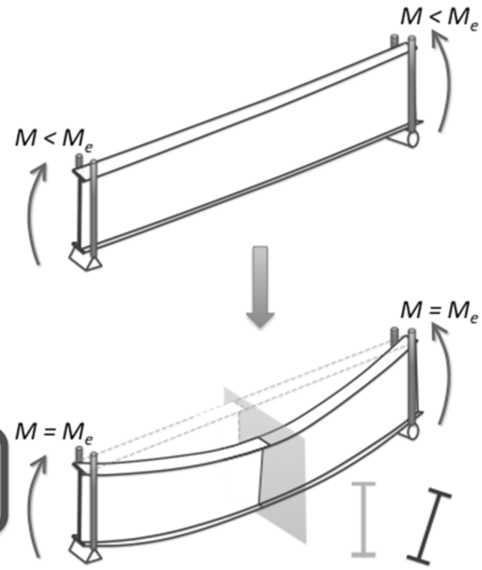
- Beam Curve – Take 3



- Where did L_p and L_r come from? (done!)
- What about those assumptions? (hmmm…)

Lateral Torsional Buckling (LTB)

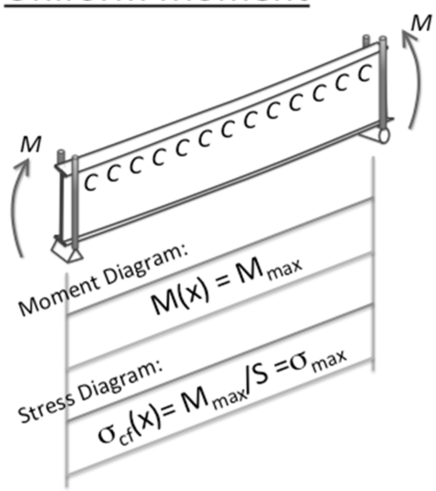
- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



Uniform Moment Distribution

- Provides for "simplest" differential equation and corresponding solution to the elastic LTB problem.
- Most conservative case
  - M(x) = constant
  - maximum compressive stress occurs along entire unbraced length
- In place of formulating and solving for other moment M(x) distributions, results can be adequately approximated by scaling the uniform moment in/elastic LTB solution.

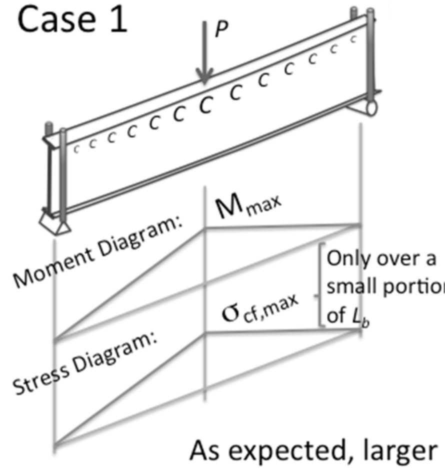
Uniform Moment



$$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

Moment gradient

Case 1

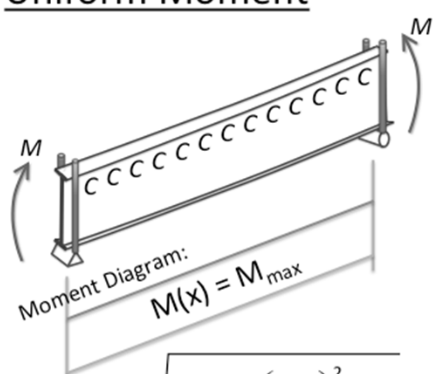


As expected, larger LTB capacity!

$M_{cr} > M_e$

$M_{cr} = C_bM_e$ with $C_b > 1.0$

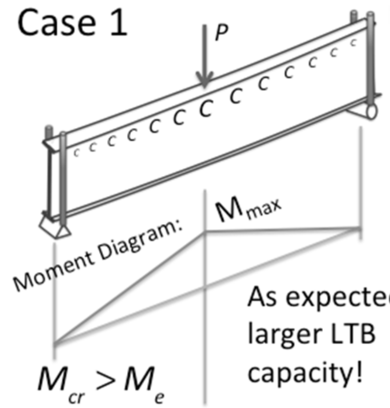
Uniform Moment



$$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

Moment gradient

Case 1



As expected, larger LTB capacity!

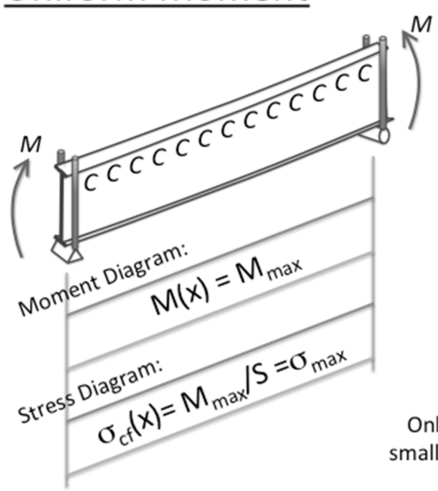
$M_{cr} > M_e$

$M_{cr} = C_bM_e$ with $C_b > 1.0$

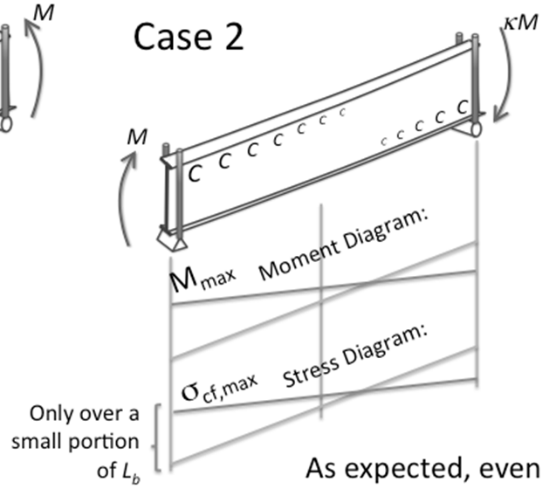
W24x68 (E=29,000, F_y = 50)
L = 300 **LBA** $\boldsymbol{C_b}$ **= ???**

Uniform Moment

Moment gradient



Case 2



$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

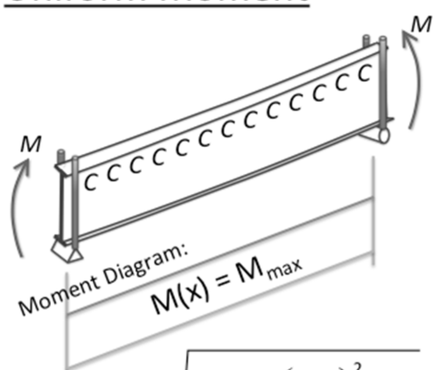
As expected, even larger LTB capacity!

$$M_{cr} > M_e$$

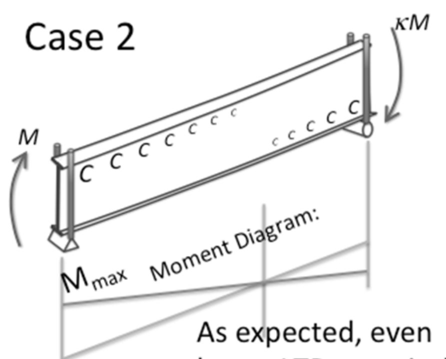
$$M_{cr} = C_b M_e \quad \text{with } C_b > 1.0$$

Uniform Moment

Moment gradient



Case 2



$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

As expected, even larger LTB capacity!

$$M_{cr} > M_e$$

$$M_{cr} = C_b M_e \quad \text{with } C_b > 1.0$$

W24x68 (E=29,000, F_y = 50)
L = 300, κ = +1 **LBA** ***C*$_b$ = ???**

LTB Moment Gradient Factor, C_b

- In/elastic LTB M_n can be adequately approximated by scaling the uniform moment in/elastic LTB solution

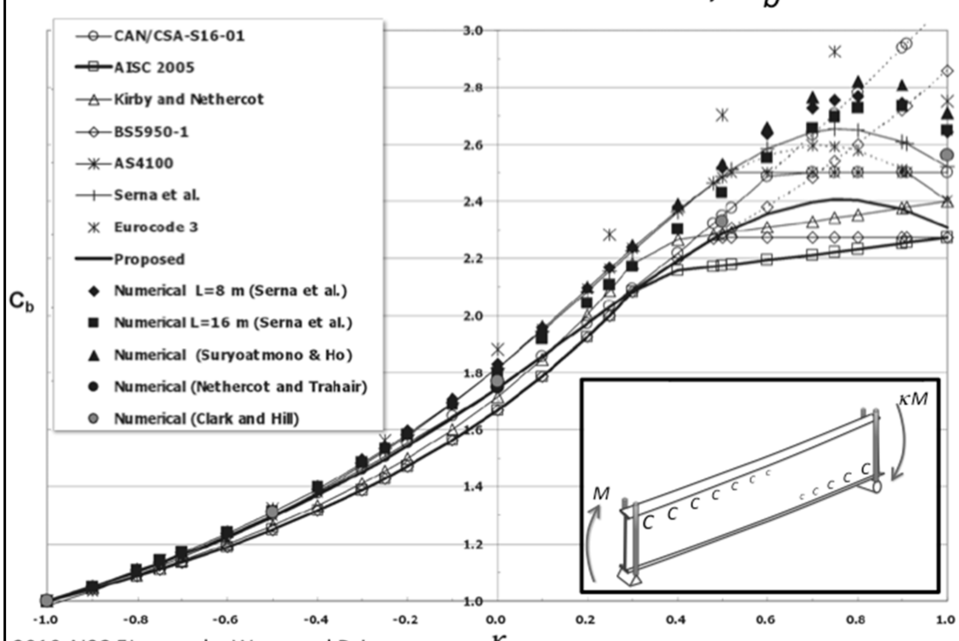
$$M_n = C_b M_n^{C_b=1} \le M_p$$

- Under no conditions can M_n exceed M_p, regardless of moment gradient
- Many possibilities for C_b, AISC uses

$$C_b = \frac{12.5\left|M_{\max}\right|}{2.5\left|M_{\max}\right| + 3\left|M_{L_b/4}\right| + 4\left|M_{L_b/2}\right| + 3\left|M_{3L_b/4}\right|}$$

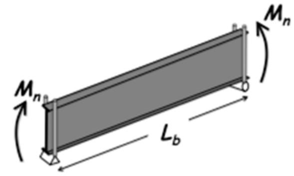
- See 2010 AISC EJ paper by Wong and Driver!

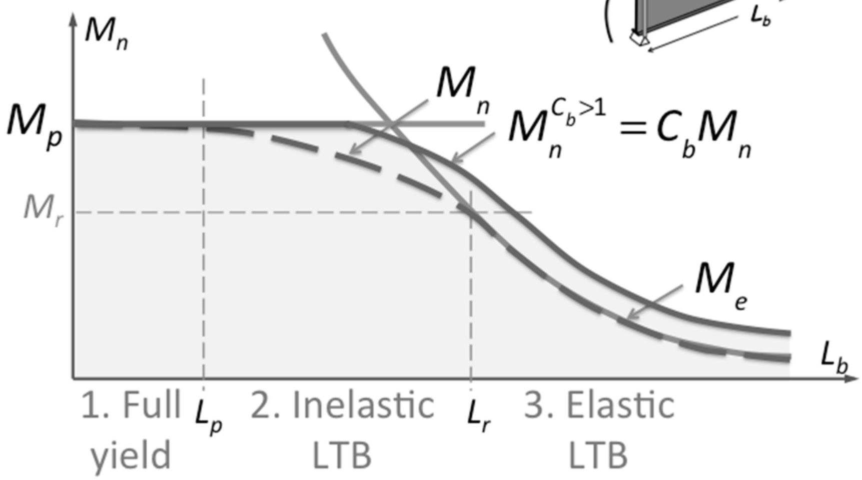
Moment Gradient Factor, C_b



2010 AISC EJ paper by Wong and Driver

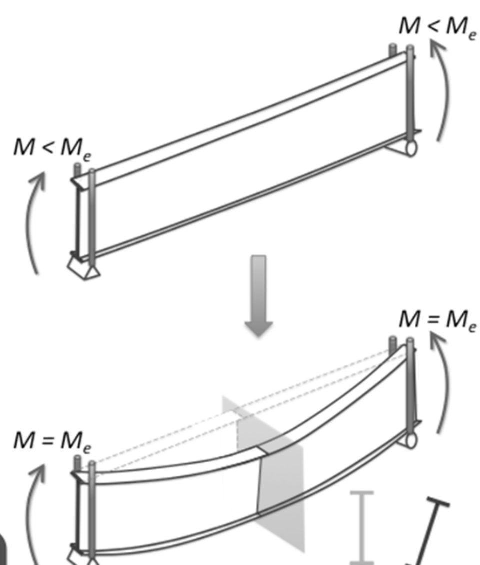
- Beam Curve – Final Take!





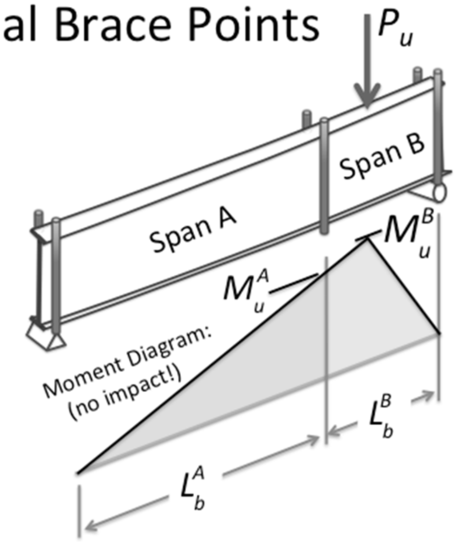
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



Providing Additional Brace Points

- Not vertical supports!
- Braces should restrain
  - twist
  - lateral movement
- All rules apply with L_b reduced to distance between brace points
- Must confirm strength within each unbraced span
- Design of braces (stiffness and strength)



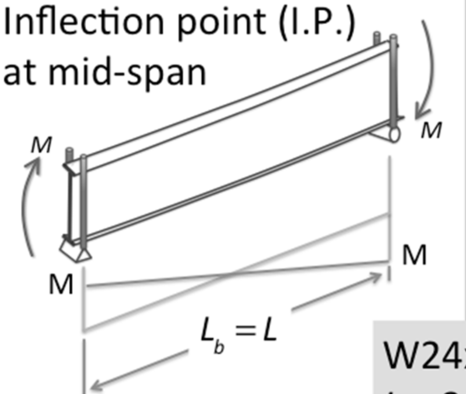
$L_b^A, C_b^A \Rightarrow M_n^A$

$M_u^A \le \phi M_n^A$

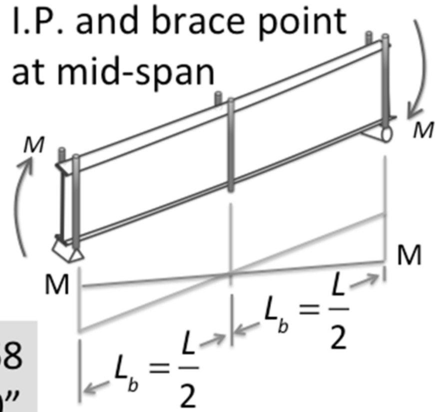
$L_b^B, C_b^B \Rightarrow M_n^B$

$M_u^B \le \phi M_n^B$

Inflection point (I.P.) at mid-span



I.P. and brace point at mid-span



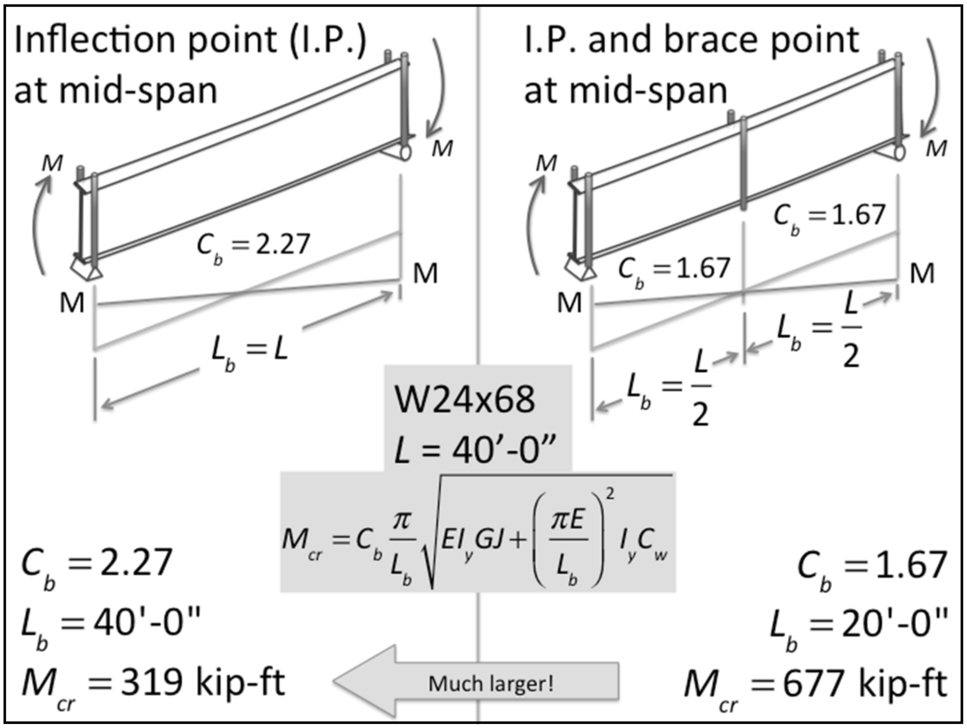
W24x68
L = 300"

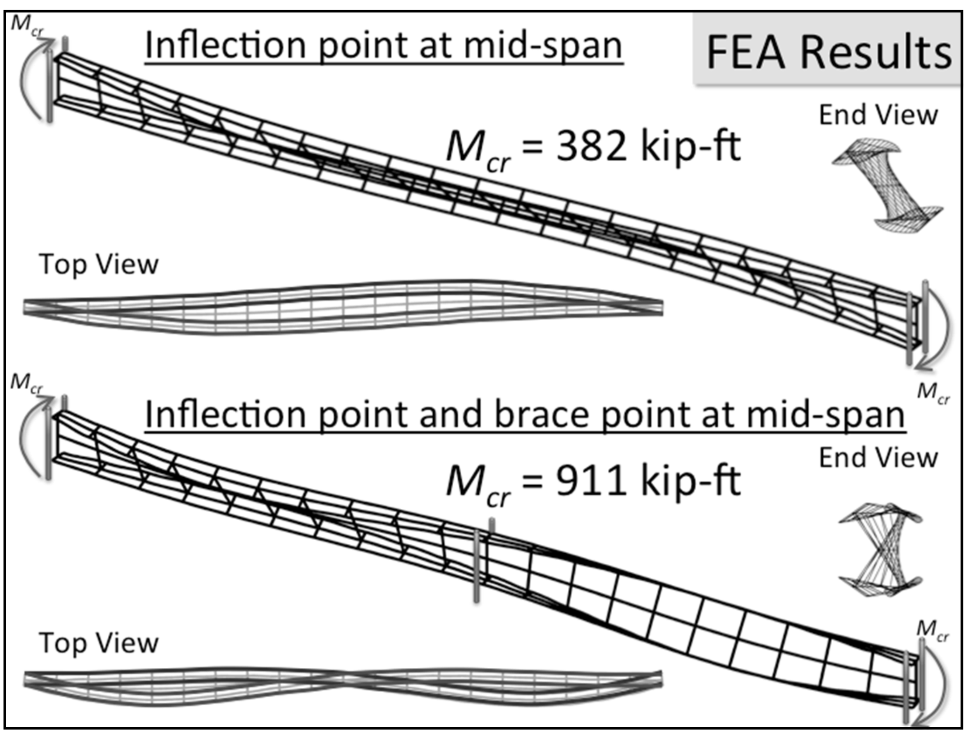
M_{cr} =

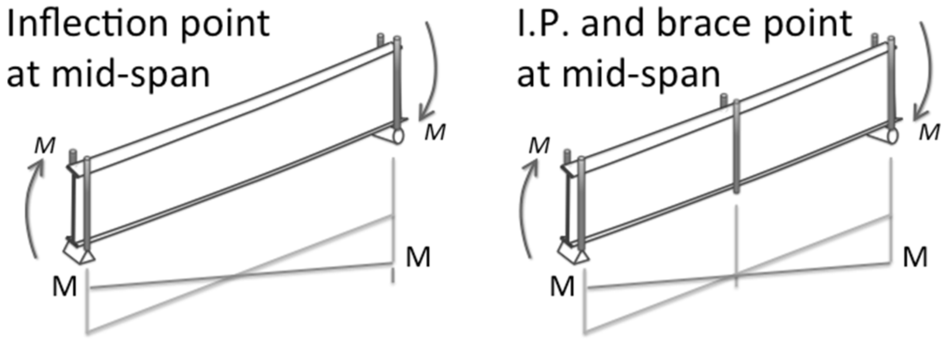
Much larger!

M_{cr} =

W24x68 (E=29,000, F_y = 50)
L = 300 **LBA**


Inflection point (I.P.) at mid-span
I.P. and brace point at mid-span
M
$C_b = 2.27$
$L_b = L$
$C_b = 1.67$
$L_b = \frac{L}{2}$
W24x68
L = 40'-0"
$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$
$C_b = 2.27$
$L_b = 40'\text{-}0"$
$M_{cr} = 319$ kip-ft
Much larger!
$C_b = 1.67$
$L_b = 20'\text{-}0"$
$M_{cr} = 677$ kip-ft


FEA Results
Inflection point at mid-span
M_{cr} = 382 kip-ft
End View
Top View
M_{cr}
Inflection point and brace point at mid-span
M_{cr} = 911 kip-ft
End View
Top View
M_{cr}



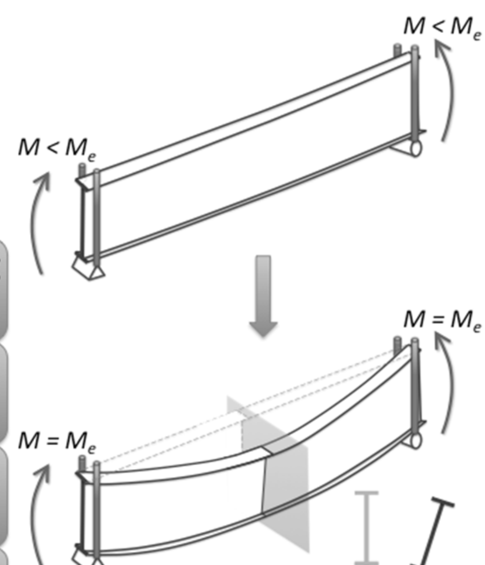
Inflection point is not a brace point!

Why does FEA give a significantly higher M_{cr} for I.P. and B.P. case? $M_{cr}^{AISC} = 677$ vs. $M_{cr}^{FEA} = 911$

Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)

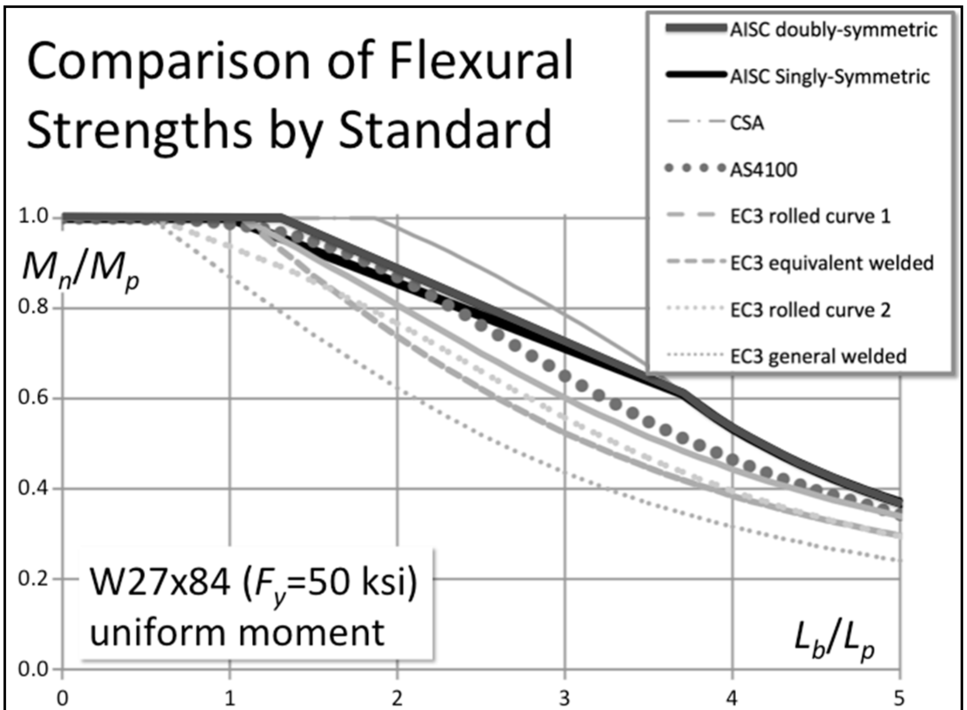
Undone
Undone
Undone
Undone



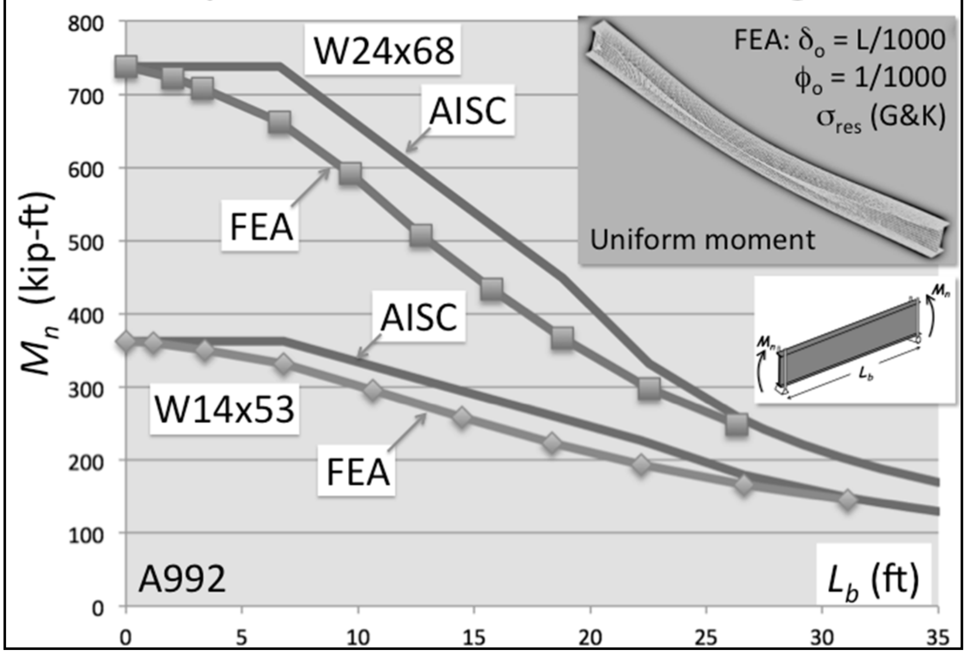
Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Behavior
  - For shorter unbraced lengths (full yielding)
  - For longer unbraced lengths (elastic LTB)
  - For intermediate unbraced lengths (inelastic LTB)

Comparison of Flexural Strengths by Standard



Comparison of Flexural Strengths



Summary

- Limit states of flexural members with focus on full yielding and lateral torsional buckling
- LTB Theory -to- Flexural Strength Beam Curve
- Beam curve accounts for:
  - full yielding
  - bending due to initial imperfection (out-of-straightness)
  - partial yielding accentuated by presence of residual stresses
  - moment gradient and brace points
- AISC, Eurocode, and other standards

Beam-columns and Structural Systems

Ron Ziemian

Lecture 7: 14-Aug-2014

PROPEC UFOP

Basis for Design of Systems

- Elastic Analysis (AISC, AS4100, Eurocode 3, …)
  - Allows for no force redistribution due to yielding
  - Strength (stability) of system is indirectly assessed by assessing strength of its components
  - In other words, strength of system is assured by ensuring adequate strength of its components
- Inelastic Analysis (AISC and AS4100)
  - Force redistribution due to yielding is accounted for in the analysis
  - System strength (stability) can be assessed directly by the analysis

Basis for Design of Systems

- Elastic Analysis (AISC, AS4100, Eurocode 3)
  - Allows for no force redistribution due to yielding
  - Strength (stability) of system is indirectly assessed by assessing strength of its components
  - In other words, strength of system is assured by ensuring adequate strength of its components

Elastic Analysis

Required Strength $\leq$ Design Strength

Stability
- Column (3 & 4)
- Beams (5 & 6)
- Beam-columns (7)

Limit States of Beam-Columns

- Full yielding (today!)
- Instability
  - Along the member length
    - Lateral-torsional buckling (Lectures 5 and 6)
    - Flexural buckling (Lectures 3 and 4)
    - Torsional-flexural buckling (today!)
  - At the cross section
    - local buckling

From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\sigma_{res} + \frac{P}{A} + \frac{M}{S} \leq \sigma_y \quad \Rightarrow \quad \frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y} \leq 1.0$$

Important Note!
No factors of safety (ϕ's or Ω's) are included in tonight's lecture…learn behavior based on nominal strengths

From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\sigma_{res} + \frac{P}{A} + \frac{M}{S} \leq \sigma_y \quad \Rightarrow \quad \frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y} \leq 1.0$$

Cross section strength (full yield):

$$\frac{P}{P_y} \geq 0.2 \qquad \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \leq 1.0$$

$$\frac{P}{P_y} < 0.2 \qquad \frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} \leq 1.0$$

From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\sigma_{res} + \frac{P}{A} + \frac{M}{S} \le \sigma_y \quad \Rightarrow \quad \frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y} \le 1.0$$

Cross section strength (full yield):

$$\frac{P}{P_y} \ge 0.2 \qquad \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \le 1.0$$

$$\frac{P}{P_y} < 0.2 \qquad \frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} \le 1.0$$

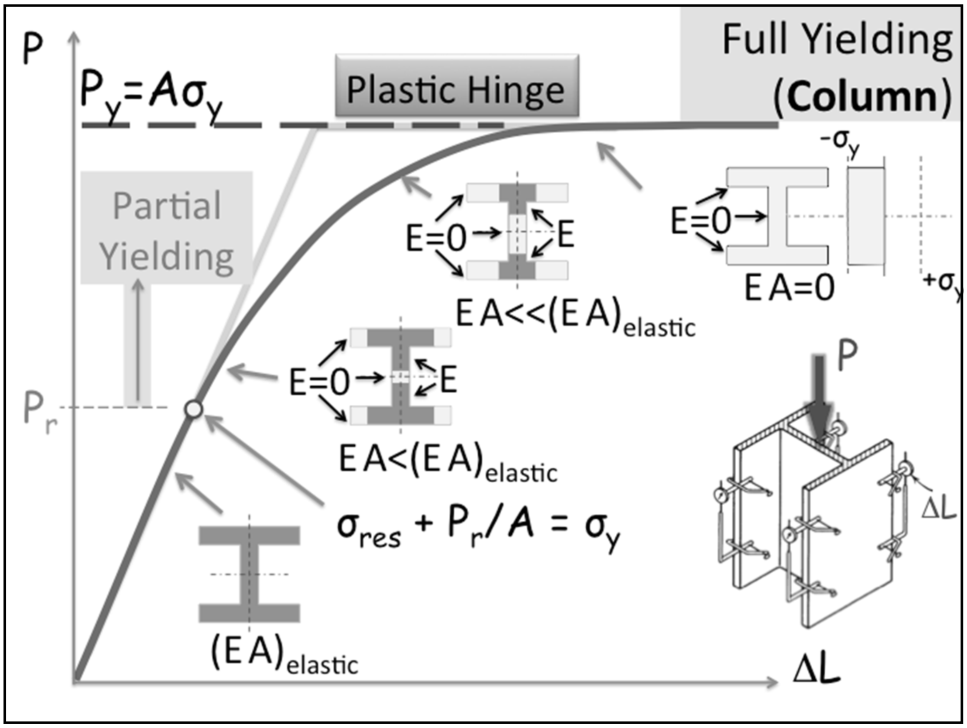
Member strength:

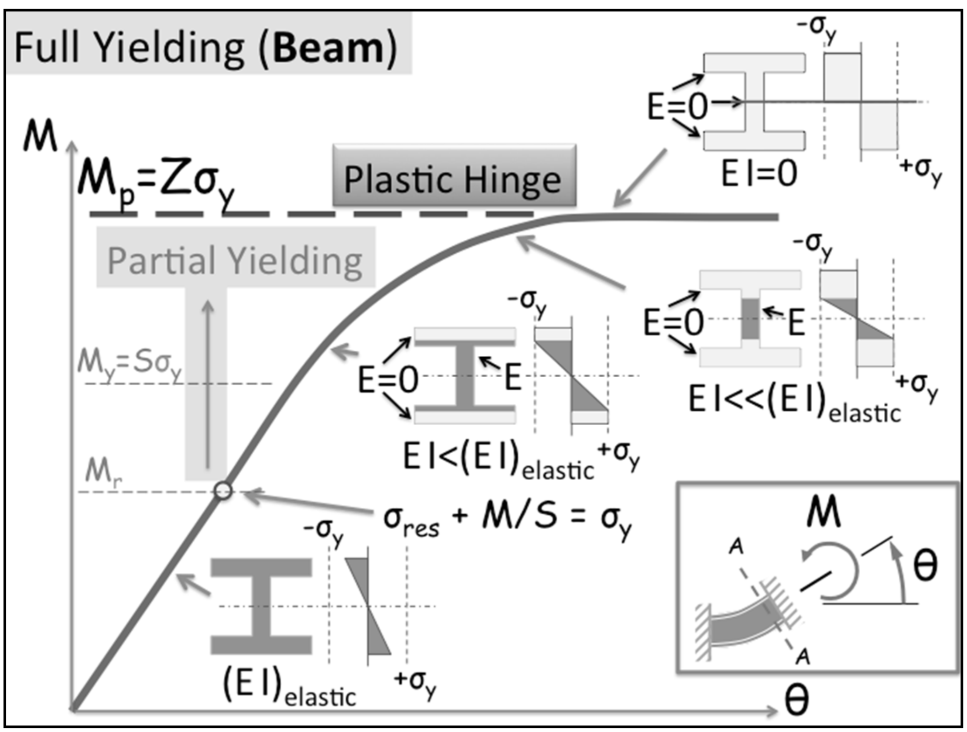
$$\frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \le 1.0$$

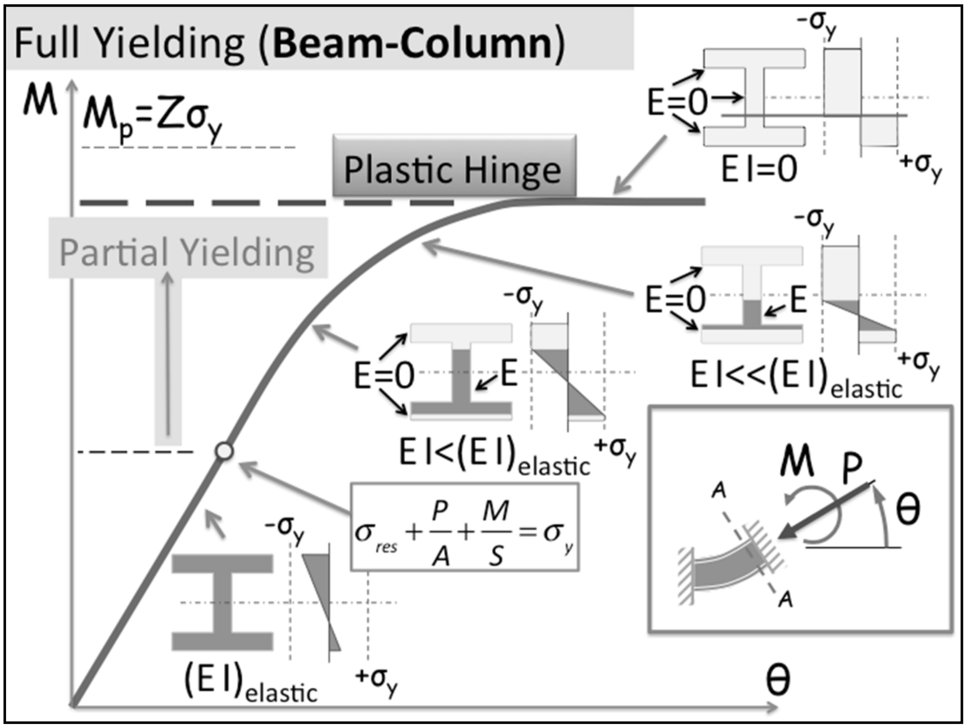
$$\frac{1}{2}\frac{P}{P_n} + \frac{M}{M_n} \le 1.0$$

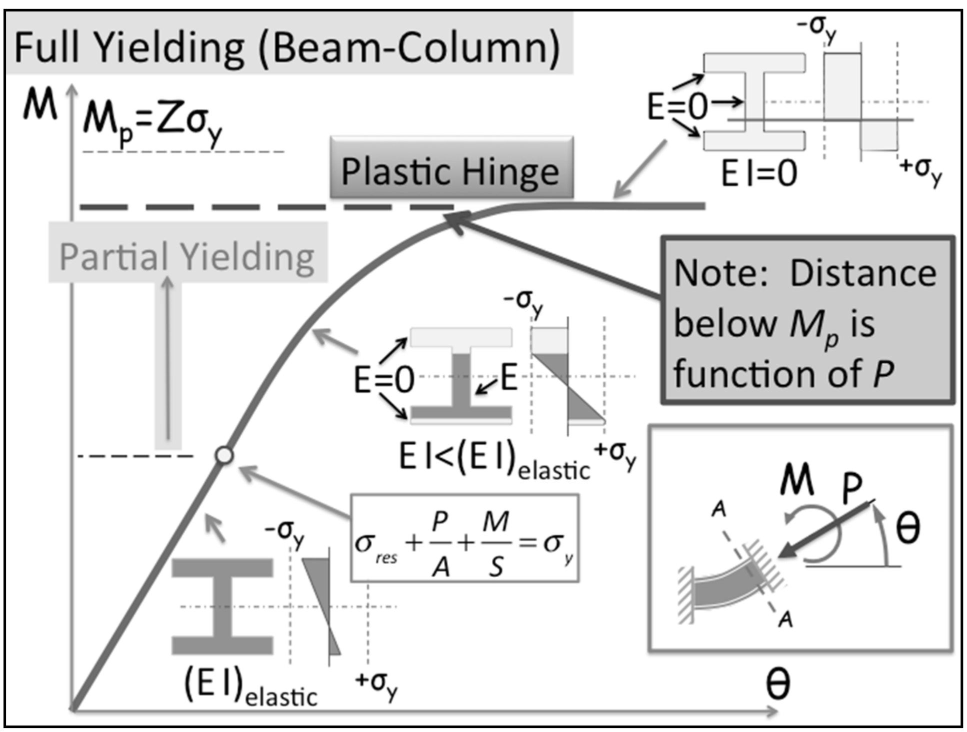
Beam-Column Strengths

- Cross section strength (small L/r)
  - full yield
  - local buckling
- Elastic member strength (large L/r)
  - compressive (P) : flexural buckling (or torsional or flexural torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- Inelastic member strength (intermediate L/r)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

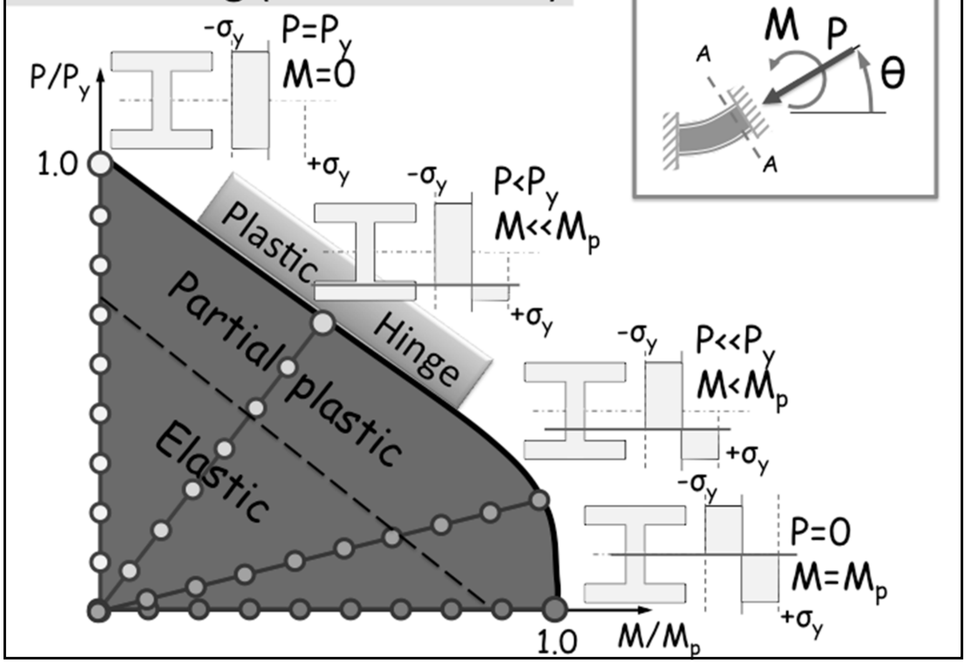

Full Yielding (Column)
P
$P_y=A\sigma_y$
Plastic Hinge
Partial Yielding
P_r
$-\sigma_y$
E=0
E
EA=0
$+\sigma_y$
EA<<(EA)elastic
EA<(EA)elastic
$\sigma_{res} + P_r/A = \sigma_y$
(EA)elastic
ΔL


Full Yielding (Beam)
M
$M_p=Z\sigma_y$
Plastic Hinge
Partial Yielding
$M_y=S\sigma_y$
M_r
$-\sigma_y$
E=0
E
EI=0
$+\sigma_y$
EI<(EI)elastic
EI<<(EI)elastic
$\sigma_{res} + M/S = \sigma_y$
(EI)elastic
A
θ

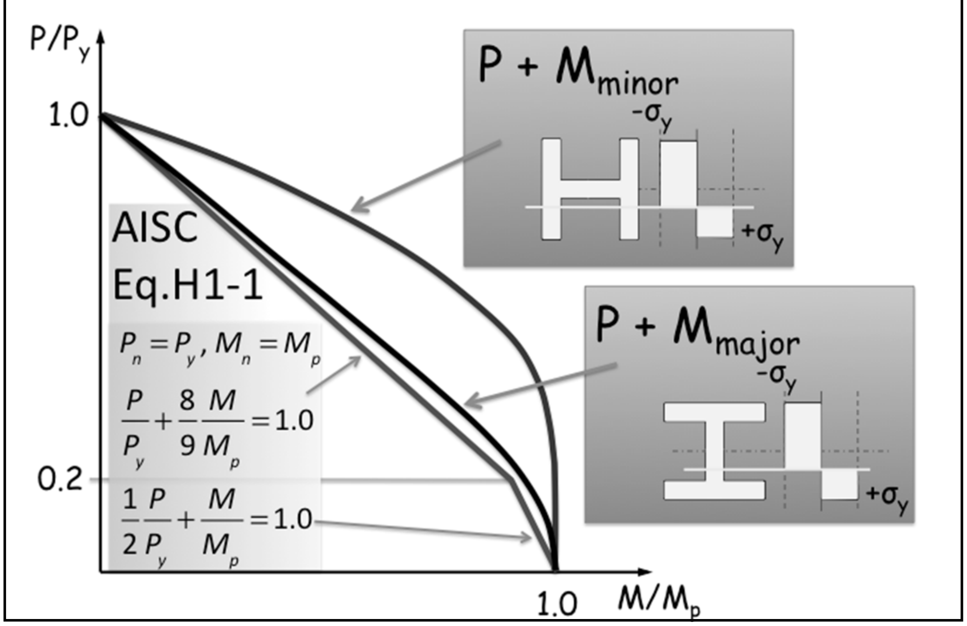

Full Yielding (**Beam-Column**)
M
$M_p=Z\sigma_y$
Plastic Hinge
Partial Yielding
E=0
E I=0
E
E I<<(E I)elastic
E I<(E I)elastic
$-\sigma_y$
$+\sigma_y$
$\sigma_{res}+\frac{P}{A}+\frac{M}{S}=\sigma_y$
(E I)elastic
M P θ
A
θ


Full Yielding (Beam-Column)
M
$M_p=Z\sigma_y$
Plastic Hinge
Partial Yielding
E=0
E I=0
Note: Distance below M_p is function of P
E
E I<(E I)elastic
$-\sigma_y$
$+\sigma_y$
$\sigma_{res}+\frac{P}{A}+\frac{M}{S}=\sigma_y$
(E I)elastic
M P θ
A
θ

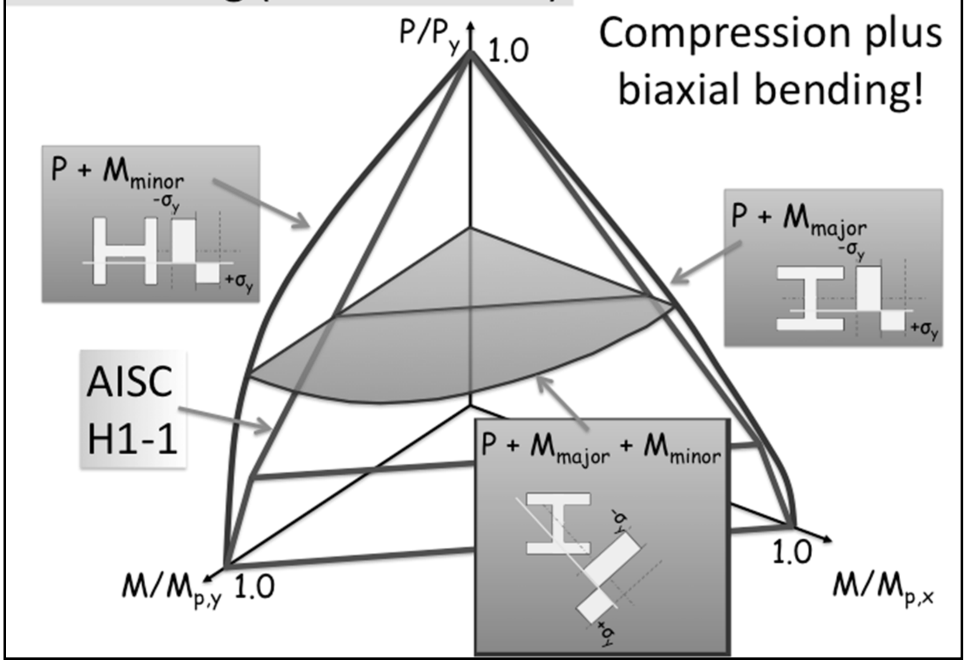
Full Yielding (Beam-Column)



Full Yielding (Beam-Column)



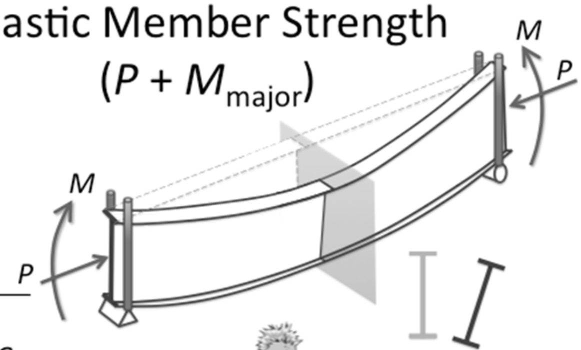
Full Yielding (Beam-Column)



Beam-Column Strengths

- Cross section strength (small *L*/*r*)
  - full yield
  - local buckling
- Elastic member strength (large *L*/*r*)
  - compressive (*P*) : flexural buckling (or torsional or flexural torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- Inelastic member strength (intermediate *L*/*r*)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

Elastic Member Strength
($P + M_{\text{major}}$)

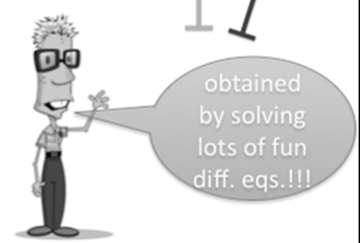


$P \neq 0$ and $M = 0$
(Session 1)

$$P_E = \pi^2 EI_y / L_b^2$$

$P = 0$ and $M \neq 0$
(Session 3)

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$



$P \neq 0$ and $M \neq 0$ (Beam-Column)

For a given M, P_{cr} is smallest root of:

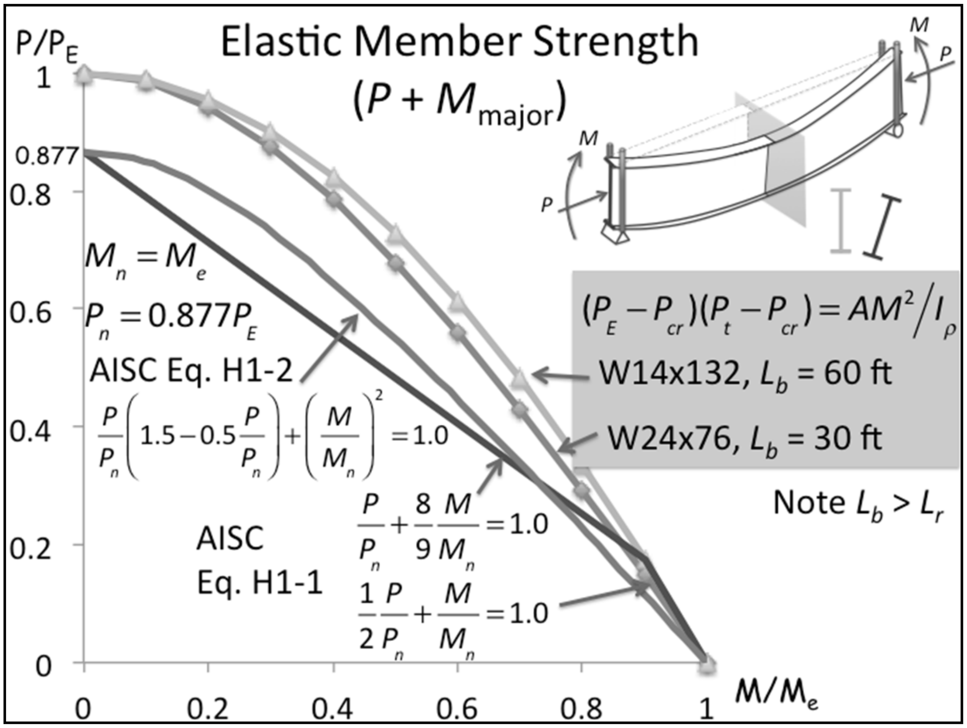
$$(P_E - P_{cr})(P_t - P_{cr}) = AM^2 / I_\rho$$

with:

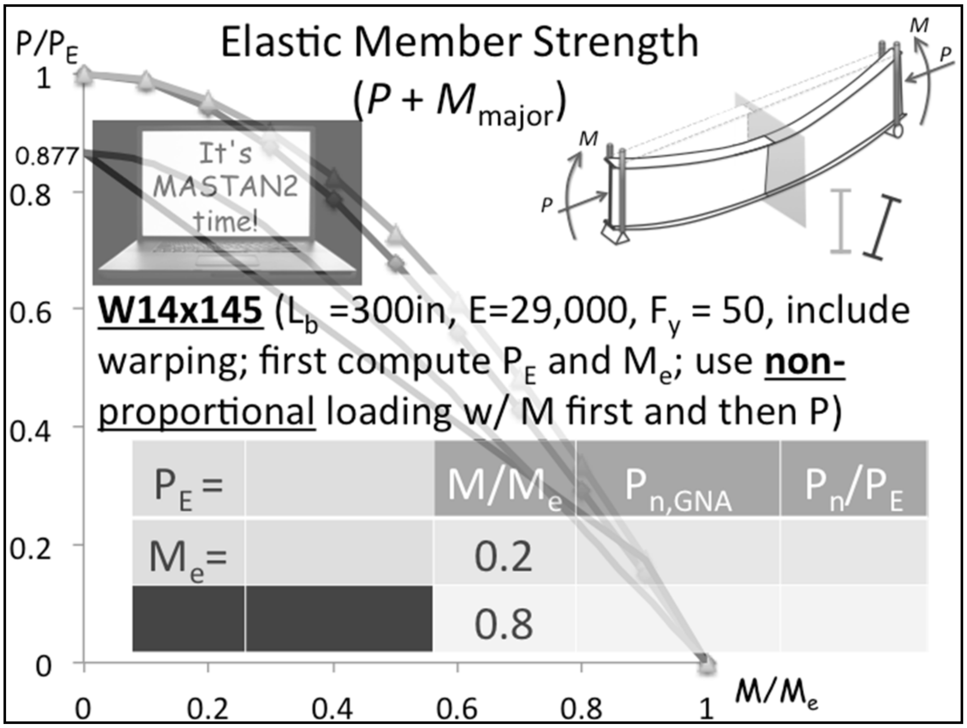
$$I_\rho = I_x + I_y$$

$$P_t = A(GJ + \pi^2 EC_w)/L^2$$

Elastic Member Strength
($P + M_{\text{major}}$)



Elastic Member Strength ($P + M_{major}$)



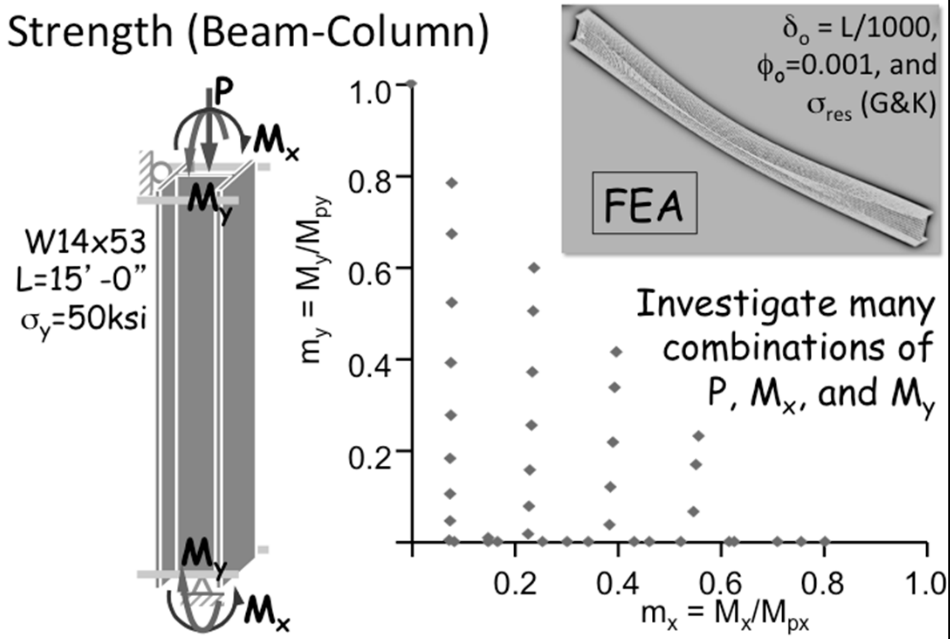
W14x145 (L_b =300in, E=29,000, F_y = 50, include warping; first compute P_E and M_e; use **non-proportional** loading w/ M first and then P)

P_E =		M/M_e	$P_{n,GNA}$	P_n/P_E
M_e=		0.2		
		0.8		

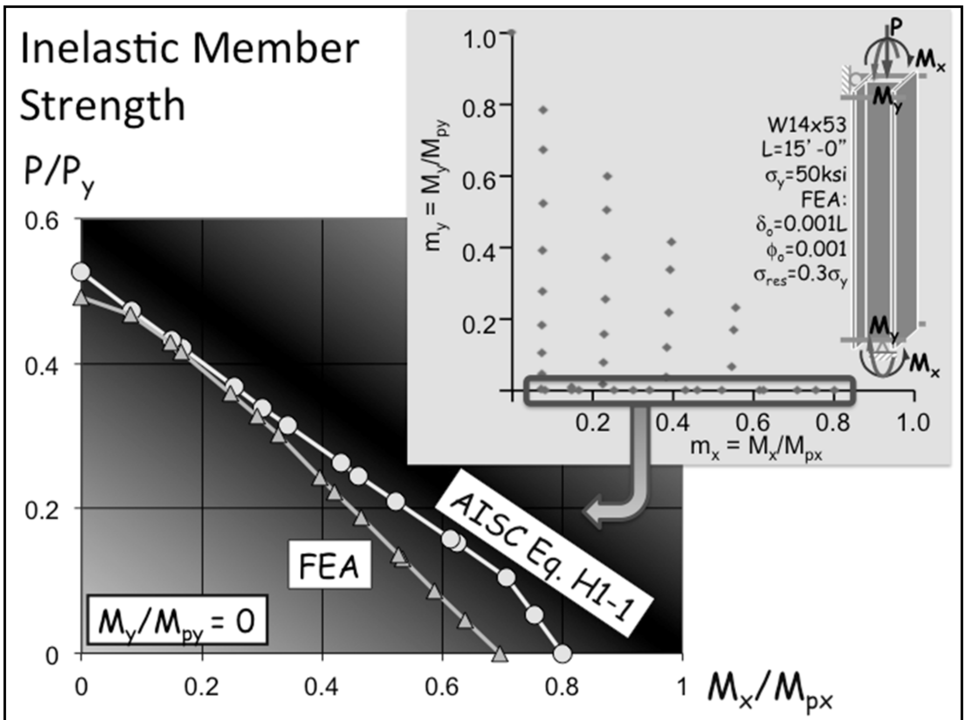
Beam-Column Strengths

- Cross section strength (small L/r)
  - full yield
  - local buckling
- Elastic member strength (large L/r)
  - compressive (P) : flexural buckling (or torsional or flexural torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- Inelastic member strength (intermediate L/r)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

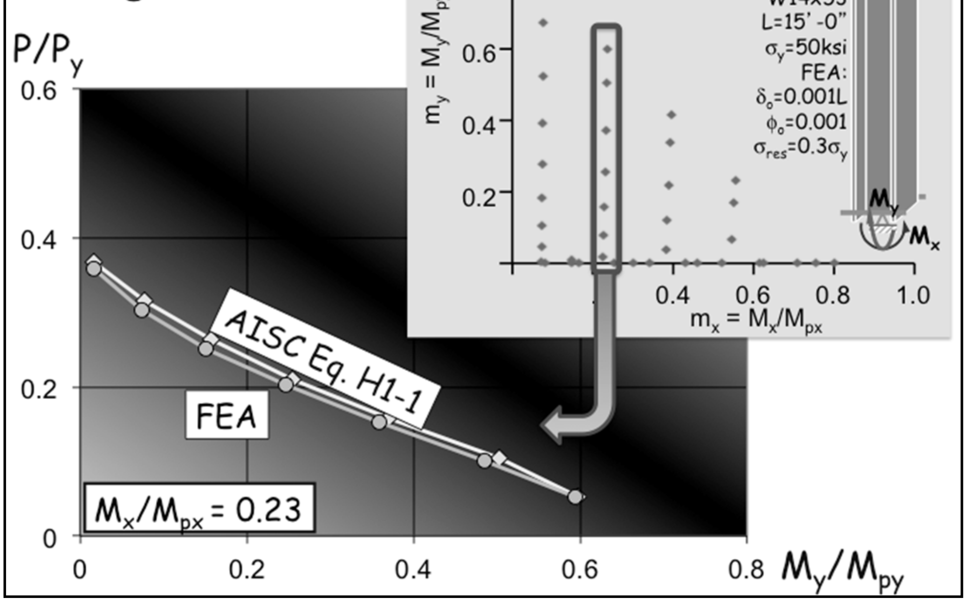
Inelastic Member Strength (Beam-Column)



Inelastic Member Strength



Inelastic Member Strength

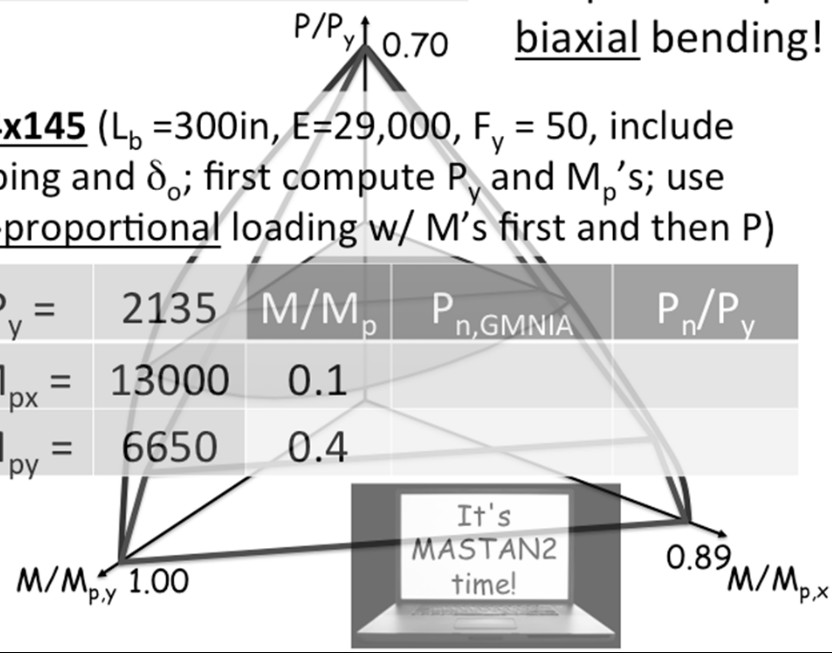


3D Beam-Column Behavior

Compression plus biaxial bending!

W14x145 (L_b =300in, E=29,000, F_y = 50, include warping and δ_o; first compute P_y and M_p's; use **non-**proportional loading w/ M's first and then P)

P_y =	2135	M/M$_p$	$P_{n,GMNIA}$	P_n/P_y
M_{px} =	13000	0.1		
M_{py} =	6650	0.4		



Analysis Essentials to obtain Required Strengths

For $\dfrac{P}{P_n} \geq 0.2, \quad \dfrac{P}{P_n} + \dfrac{8}{9}\dfrac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its components (AISC, Ch C – *Design for Stability*):

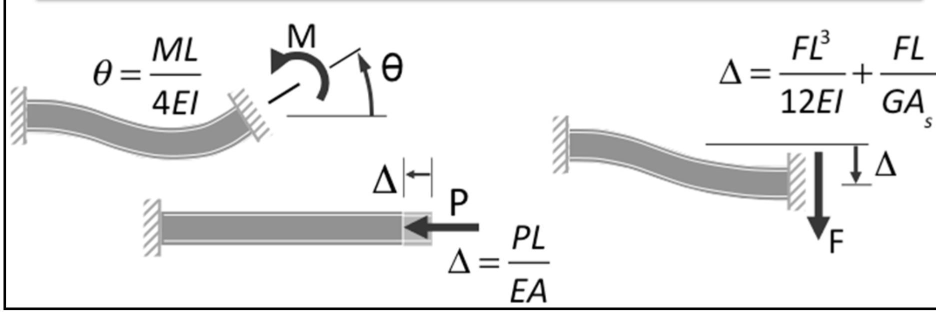
- flexural, shear, axial deformations
- second-order effects (both P-Δ and P-δ)
- geometric imperfections
- stiffness reductions due to inelasticity

Analysis Essentials to obtain Required Strengths

For $\dfrac{P}{P_n} \geq 0.2, \quad \dfrac{P}{P_n} + \dfrac{8}{9}\dfrac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its components (AISC, Ch C – *Design for Stability*):

- flexural, shear, axial deformations



Analysis Essentials to obtain Required Strengths

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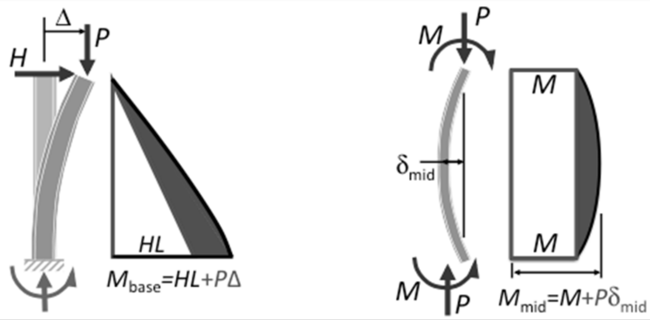


Including Second-Order Effects

- Analysis options for :
  - Rigorous analysis (recommended!)
    - loads applied incrementally/iteratively
    - geometric stiffness matrix or use of stability functions
    - updating geometry after each increment of loading
  - Approximate amplification factors
    - *$M = B_1M_{nt} + B_2M_{lt}$ (AISC)*
    - *Eurocode has similar equations…*
- Approximate amplifiers can be useful indicators of the significance of
  - $P\delta$ effects
  - $P\Delta$ effects

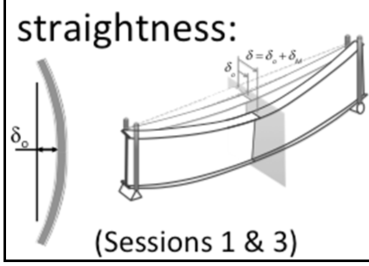
Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its components (AISC, Ch C – *Design for Stability*):

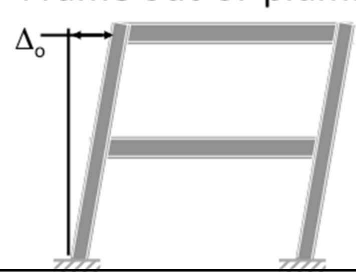
- geometric imperfections

Member out-of-straightness:



(Sessions 1 & 3)

Frame out-of-plumb:



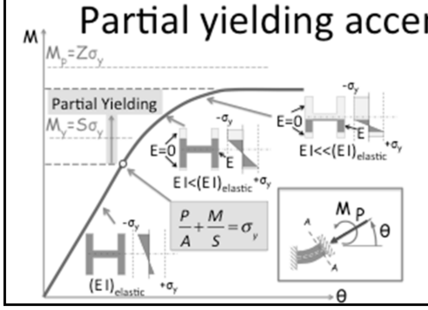
Analysis Essentials to obtain Required Strengths

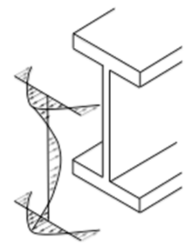
For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its components (AISC, Ch C – *Design for Stability*):

- stiffness reductions due to inelasticity

Partial yielding accentuated by residual stresses:





Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

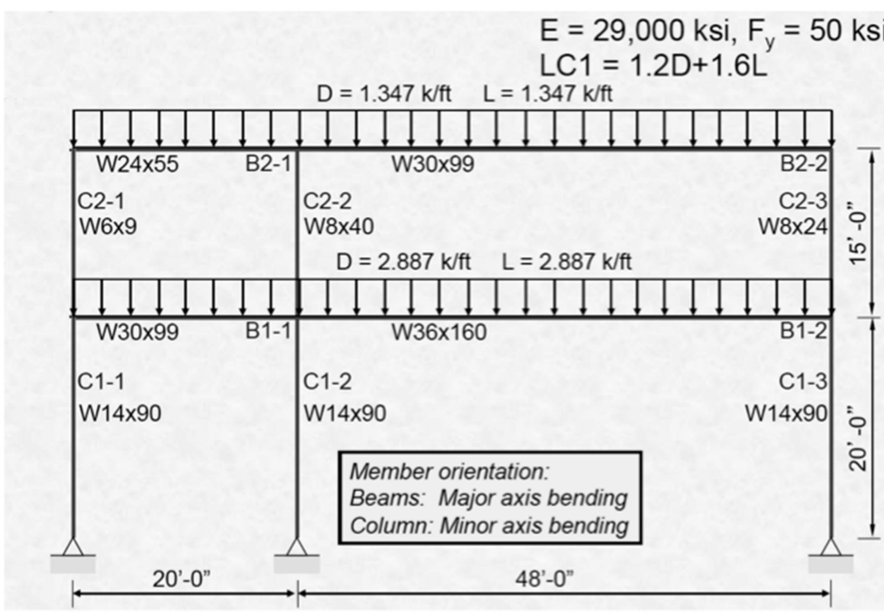
Effects that may impact stability of system and its components (AISC, Ch C – *Design for Stability*):

- flexural, shear, axial deformations ✔
- second-order effects (both *P*-Δ and *P*-δ) ✔
- geometric imperfections ✔
- stiffness reductions due to inelasticity ✔

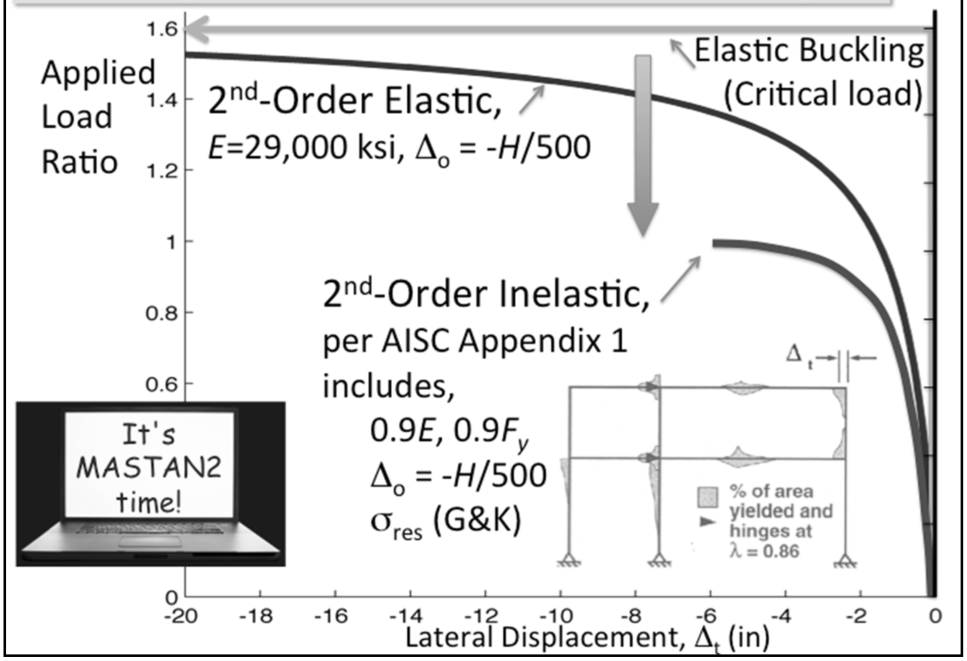
Methods for Designing for Stability

- Any rational method that considers all of the effects
- Methods appearing in AISC Specification
  - Based on 2nd-order Elastic Analysis
    - Effective Length or Buckling Length Method
      - Established in early 1960's
    - Direct Analysis Method
      - Established in AISC in 2005
  - Based on 2nd-order Inelastic Analysis
    - Appendix 1

System Strength (Stability) – Frame Example



Predicting "actual" behavior with elastic analysis:

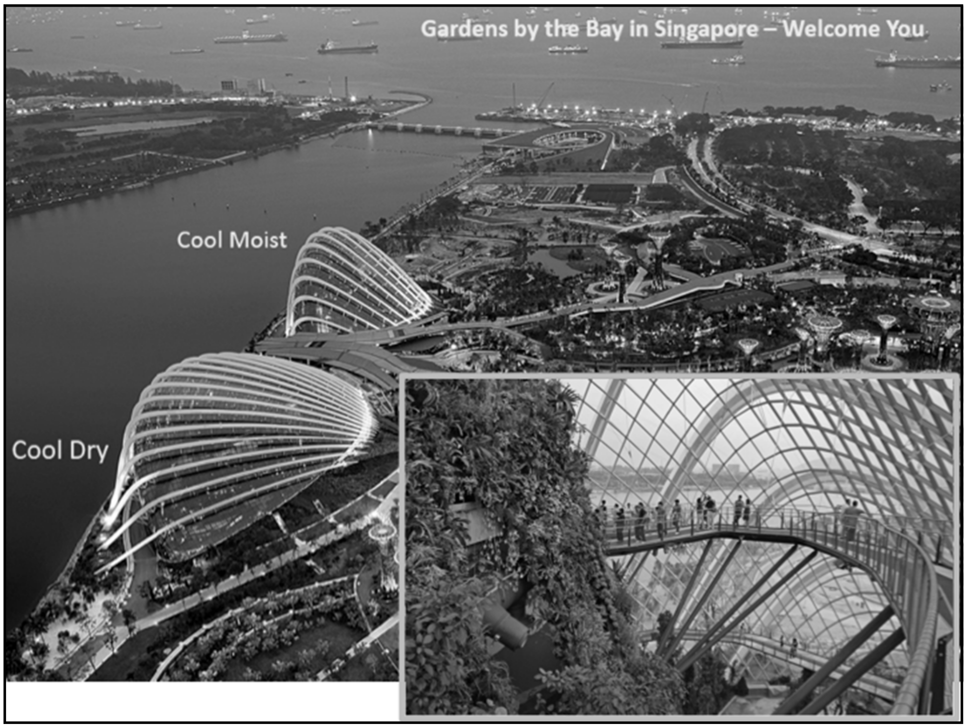


From: Gmail <chenwilfred@gmail.com>
Subject: Fwd: Second order analysis and design of flower dome and cloud forest conservatories - Structural Excellent Award 2013
Date: May 13, 2014 1:41:44 PM EDT
To: Guo-Qiang Li <gqli@tongji.edu.cn>, Bjorhovde Reidar <rbj@bjorhovde.com>, Galambos Ted <galam001@umn.edu>, ziemian Ron <ziemian@bucknell.edu>, Zandonini Riccardo <Riccardo.Zandonini@ing.unitn.it>, Bazant Zdenek <z-bazant@northwestern.edu>, Jerry lu <iu.jerryu@gmail.com>

The two structures they designed are beautiful and nature with advanced analysis.
W.F.Chen

Begin forwarded message:

> **From:** "Liew Jat Yuen, Richard" <ceeljy@nus.edu.sg>
> **Date:** May 11, 2014 at 11:31:21 PM PDT
>
> **Subject: Second order analysis and design of flower dome and cloud forest conservatories - Structural Excellent Award 2013**
>
> Dear Prof Chen
> To share with you that Prof S L Chan and I have design this unusual structures in Singapore using the direct second order analysis theory.
> The use of effective length method in this project is not valid and we adopt the direct second order method to check the design of this awarded winning project.
>
> We have been using this new design approach to design several iconic and unusual structures in Singapore and Hong Kong.
>
> With regards
> **Richard Liew**
> Professor, National University of Singapore
> Department of Civil & Environmental Engineering



Summary

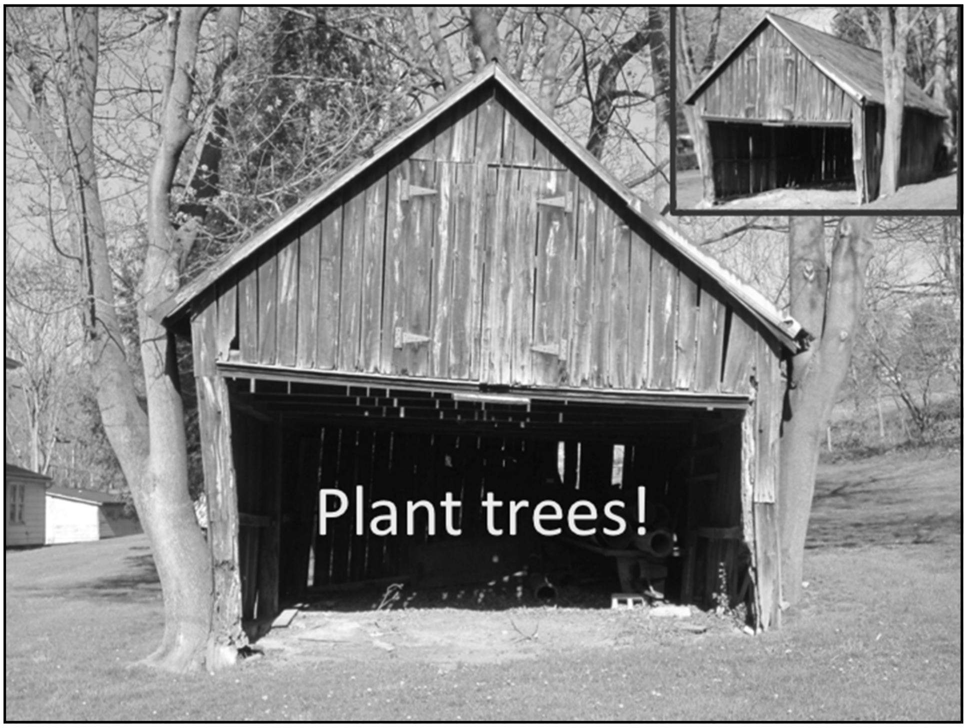
- Basis for Design of Systems
  - Elastic Analysis
    - strength of system is assured by ensuring adequate strength of its components
  - Inelastic Analysis
    - System strength (stability) can be assessed directly by the analysis
- Stability of members
  - Compression (Lectures 3 & 4)
  - Flexural (Lectures 5 & 6)
  - Combined compression and flexure (Lecture 7)
    - Behavior of beam-columns (today!)
    - Behavior of Systems (only a small amount, today!)

Summary (2)

- Behavior of Beam-Columns
  - full yield (interaction surface)
  - member instability
    - from flexural buckling -to- lateral torsional buckling, including torsional-flexural buckling
- Factors impacting system stability
  - flexural, shear, axial deformations
  - second-order effects
    - rigorous analysis vs. amplification factors
  - geometric imperfections
  - stiffness reductions due to inelasticity

Summary (3)

- Design Systems for Stability
  - elastic analysis vs. inelastic analysis
- Elastic analysis
  - Effective or buckling length method ($KL > L$)
  - Direct analysis method ($KL = L$)
  - Discussion on comparison
- Finally, if you still do not have confidence that your structural system is stable, then you can always…



Applying Nonlinear Analysis to Learn the Fundamentals of Structural Stability

Ron Ziemian

13-Aug-2014 to 14-Aug-2014

Course Overview

- Employ a virtual laboratory to learn basic concepts of structural stability
- Seven 90-minute lectures
  - Lectures 1 & 2 Introduction to Nonlinear Analysis
  - Lectures 3 & 4 Behavior of Compression Members
  - Lectures 5 & 6 Behavior of Flexural Members
  - Lecture 7 Beam-columns and Structural Systems
- Software employed is MASTAN2 which is available at no cost at www.mastan2.com

Better designs will come from a better understanding of behavior

G. Winter, W. McGuire, T. Pekoz, A. Nilson, J. Abel, P. Gergely, R. White, T. Ingrafea

$$U = \sum_{i=1}^{\infty} Experiences_i$$

American Iron and Steel Institute

The Aluminum Association

+

PROPEC UFOP

Obrigado !